Applying Theorem 1 to F, we obtain

(10) $$T(r, f) = T(r, F) + O(1) < 3\,N(r, 1/F) + 4\,\overline{N}(r, 1/(\psi_F - 1)) + S(r, F) = 3\,N(r, 1/(f - w_1)) + 4\,\overline{N}(r, 1/(\psi_f - w_2)) + S(r, f)\,.$$

If $f - w_1$ and $\psi_f - w_2$ have both only a finite number of zeros it follows from (10) and (2) that

$$\{1 + o(1)\}\; T(r, f) = O(\log r)$$

as $r \to \infty$ outside a set of finite measure.

This implies that

$$\liminf_{r \to \infty} \frac{T(r, f)}{\log r} < \infty\,,$$

so that f is a rational function contrary to our hypothesis that f is transcendental. This proves (i).

On the other hand, if w_1 is an evB for f and w_2 is an evB for ψ_f for distinct zeros then we can choose a positive number $\lambda < \varrho$, where ϱ is the order of f, such that

$$N(r, 1/(f - w_1)) = O(r^\lambda) \quad \text{and} \quad \overline{N}(r, 1/(\psi_f - w_2)) = O(r^\lambda)\,.$$

Choosing μ such that $\lambda < \mu < \varrho$, we then have

(11) $$\int_{r_0}^{\infty} \frac{N(x, 1/(f - w_1))}{x^{1+\mu}}\,\mathrm{d}x < \infty \quad \text{and} \quad \int_{r_0}^{\infty} \frac{\overline{N}(x, 1/(\psi_f - w_2))}{x^{1+\mu}}\,\mathrm{d}x < \infty\,.$$

Also, by (1),

$$\int_{r_0}^{r} \frac{S(x, f)}{x^{1+\mu}}\,\mathrm{d}x = o\left(\int_{r_0}^{r} \frac{T(x, f)}{x^{1+\mu}}\,\mathrm{d}x\right).$$

Hence, by (10),

$$\{1 + o(1)\} \int_{r_0}^{r} \frac{T(x, f)}{x^{1+\mu}}\,\mathrm{d}x \leqq 3 \int_{r_0}^{r} \frac{N(x, 1/(f - w_1))}{x^{1+\mu}}\,\mathrm{d}x + 4 \int_{r_0}^{r} \frac{\overline{N}(x, 1/(\psi_f - w_2))}{x^{1+\mu}}\,\mathrm{d}x\,,$$

whence it follows by (11) that

$$\int_{r_0}^{\infty} \frac{T(x, f)}{x^{1+\mu}}\,\mathrm{d}x < \infty\,.$$

This implies that ϱ = the order of $f \leqq \mu$, which is a contradiction. This proves (ii) and completes the proof of Theorem 2.

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