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<u>Digizeitschriften e.V.</u> SUB Göttingen Platz der Göttinger Sieben 1 37073 Göttingen the latter inscribed into the former. The six sides, ijk, ilm, ikl, imj, ijl, ikm, of every quadrangle pass, respectively, through the six vertices jk, lm, kl, jm, jl, km, of the quadrilateral.

The same Desargues configuration reveals six pairs of mutually inscribed pentagons. The five vertices ij, jk, kl, lm, mi, of one lie on the five sides jli, lik, ikm, kmj, mjl of the other.

By placing the vertices of a pair of pentagons appropriately we get a self-inscribed decagon and by the use of 24 permutations of A, B, C, D (where A, B, C, D = i, j, k, l, m), we get 24 self-inscribed decagons as given below.

Writing 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 for ij, jl, jk, km, kl, il, lm, jm, im, ik, respectively we get the decagons

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(1). 0 1 2 3 4 5 6 7 8 9,
                                  (2). 0 7 2 9 4 1 6 3 8 5,
 (3). 0 1 4 5 8 9 2 3 6 7,
                                  (4). 0 9 4 3 8 7 2 1 6 5,
 (5). 0 2 1 6 4 9 3 7 8 5,
                                  (6). 0 7 1 5 4 2 3 6 8 9,
 (7). 0 2 4 9 8 5 1 6 3 7,
                                  (8). 0 5 4 6 8 7 1 2 3 9,
 (9). 0 2 7 6 3 9 4 1 5 8,
                                 (10). 0 1 7 8 3 2 4 6 5 9,
(11). 0 2 3 9 5 8 7 6 4 1,
                                 (12). 0 8 3 6 5 1 7 2 4 9,
(13). 0 5 9 3 4 1 6 8 7 2.
                                 (14). 0 8 9 2 4 5 6 3 7 1,
(15). 0 5 4 1 7 2 9 3 6 8,
                                 (16). 0 2 4 3 7 8 9 5 6 1,
(17). 0 7 2 4 3 8 6 1 5 9,
                                 (18), 0 1 2 9 3 7 6 4 5 8,
(19). 0 7 3 8 5 9 2 4 6 1,
                                 (20). 0 9 3 4 5 1 2 7 6 8,
(21). 0 7 1 4 6 8 3 2 9 5,
                                 (22). 0 2 1 5 6 7 3 4 9 8,
(23). 0 7 6 8 9 5 1 4 3 2,
                                 (24). 0 5 6 4 9 2 1 7 3 8.
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The group G of symmetries of the Desargues figure which leaves both the set P of 10 points and the set L of 10 lines invariant is the group of permutations of the 10 points, which preserve the set L of lines. If we also allow the reciprocity which interchanges P and L, we obtain the group G'. It has been shown by Coxeter (1950) that G has order 120 while G' has order 240. It is clear from this paper that G contains a subgroup of order 24, the symmetric group of degree four on A, B, C, D, (where A, B, C, D = i, j, k, l, m).

There are several ways in which G can be calculated. G is transitive over 10 points, 10 lines, the 24 self-inscribed decagons.

The stabilizers G_P (of a point), G_1 (of a line), G_d (of a self-inscribed decagon) have orders 12, 12, and 5 respectively. Hence G has order $10 \times 12 = 10 \times 12 = 24 \times 5 = 120$.

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Reference

Coxeter, H. S. M., Self-dual configurations and graphs, Bulletin of the American Mathematical Society, 56, 413-455; (1950).

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