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Proof. Assume that the family $\mathscr{F}$ is dense in $\mathscr{G}$. For any mapping $g \in \mathscr{G}$ there exists a transfinite sequence $\{f_\xi\}_{\xi<\Omega}$ of mappings $f_\xi \in \mathscr{F}$ such that $g = \lim_{\xi\to\Omega} f_\xi$. By Lemma 1 there exist among them such mappings f_ξ which are extensions of $g \mid A$. Hence the condition is necessary.

Assume now that the condition is satisfied. Assume, as in the proof of Theorem 2, that $E = \bigcup_{\xi\to\Omega} A_\xi$ where A_ξ are denumerable sets and $\xi < \zeta$ implies $A_\xi \subset A_\zeta$. Let $g \in \mathscr{G}$. According the assumption there exists for every set A_ξ a mapping $f_\xi \in \mathscr{F}$ such that $f_\xi \mid A_\xi = g \mid A_\xi$. Hence follows as was the case with Theorem 2 that $g = \lim_{\xi<\Omega} f_\xi$. The family $\mathscr{F}$ is therefore dense in $\mathscr{G}$. The condition proves to be sufficient.

Exemple 3. Let $\mathscr{A}$ be the family of all functions approximatively continuous defined on R. Let $\mathscr{B}_1$ be the family of all Baire class 1 functions. It follows from the continuum hypothesis that $\mathscr{A}$ is dense in $\mathscr{B}_1$. In fact, G. Petruska and M. Laczkovich have demonstrated in [6] that for any function $g \in \mathscr{B}_1$ and for every set A of measure zero (and therefore also for any denumerable set) there exists a function $f \in \mathscr{A}$ such that $f \mid A = g \mid A$. This implies by Theorem 3 the denseness of $\mathscr{A}$ in $\mathscr{B}_1$.

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