for each $z \in [\alpha, \beta] \cap Q$ and $f(z) = 0$ otherwise. Then $f \in G_0$ and hence $2^{1-n} \in F_x$. Thus by (i), 2^{1-n} is a generator of the group F_x and therefore F_x is cyclic. Hence each subgroup of F_x is cyclic; by Lemma 2, H is cyclic.

Lemma 4. *Let $0 < f \in G_0$. Then f is not singular.*

Proof. Suppose that f is singular. Then each $f_1 \in G_0$, $0 < f_1 < f$ is singular. There exist irrational numbers α_1, β_1 and a real $c \neq 0$ such that $f(x) = c$ for each $x \in [\alpha_1, \beta_1] \cap Q$. Let $f_1 \in G_0$ such that $f_1(x) = f(x) = c$ for each $x \in Q \cap [\alpha_1, \beta_1]$ and $f_1(x) = 0$ otherwise. Clearly $f_1 \in G$ and $0 < f_1 \leqq f$. Let

$$N_1 = \{\varphi^{-1}(x) : x \in Q \cap [\alpha_1, \beta_1]\}.$$

Let k be the least element of N_1. According to (i) and (ii), $2^{k-1}c$ is an integer. We can choose irrational numbers $\alpha < \beta$ such that $[\alpha, \beta] \subset [\alpha_1, \beta_1]$ and $\varphi(k) \notin [\alpha, \beta]$. Let $y \in [\alpha, \beta] \cap Q$. Put $\varphi^{-1}(y) = t$. Since $t > k$, we infer that $2^{k-1}(\frac{1}{2}c)$ is an integer. Thus the function $g \in G_0$ defined by

$$g(x) = \tfrac{1}{2}c \quad \text{if} \quad x \in [\alpha, \beta] \cap Q \quad \text{and} \quad g(x) = 0 \quad \text{otherwise}$$

belongs to G_0. We have $0 < 2g < f_1$, hence $g < f_1 - g$ and therefore

$$g \wedge (f_1 - g) = g > 0;$$

thus f_1 cannot be singular. This shows that f is not singular.

From Lemma 2 and Lemma 4 it follows that there exists an archimedean lattice ordered group fulfilling (b) with no singular elements.

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