divisors of m.) Thus, there are at most $d^2(x_3)\, d(a_1 d_1 + a_2 d_2) \ll f^3(E)$ values of p for which (6) is solvable, where $f(n)$ is the maximum value of $d(k)$ for all $k \leqq n$.

Case II. Suppose p divides only one of the integers x_1 and x_2. Say $x_1 = py_1$ and $(x_2, p) = 1$. Then

$$\frac{kx_3 - pa_3}{px_3} - \frac{a_1}{py_1} = \frac{y_1(kx_3 - pa_3) - x_3 a_1}{px_3 y_1} = \frac{a_2}{x_2},$$

which implies $p \mid y_1 k x_3 - x_3 a_1$ and so $p \mid y_1 k - a_1$. By Theorem 1, $x_1 = px_3(a_1 d_1 + a_2 d_2)/kd_1$ where $d_i \mid x_3 p$ which implies $y_1 < E^3/k^2$ and so $y_1 k - a_1 < E^3$. Hence, there are at most $d(y_1 k - a_1) < f(E^3)$ values of p for which (6) solvable.

Now by [2, Theorem 317] $f(n) = O(\exp(\log n/\log\log n))$, and so both $f^3(E)$ and $f(E^3)$ are $O(\exp(3c \log k))$. Therefore, the total number of primes p, $E \leqq p \leqq 2E$ for which (5) is solvable, is $O(\exp(3c \log k)\, E/k)$. However, there are at least $E/\log^2 k$ primes between E and $2E$, and hence picking $c < 1/3$ we see that there must be some primes $> E$ for which (5) is unsolvable.

Corollary. *There is a constant $c > 0$ such that*

$$\lambda'(k; a_1, a_2, a_3) > \exp(c \log k \log\log k)$$

for all k sufficiently large.

Proof. By the above argument, it is clear that there exist primes p, $E \leqq p \leqq 2E$ such that all eight equations

$$\frac{k}{p} = \frac{\pm a_1}{x_1} + \frac{\pm a_2}{x_2} + \frac{\pm a_3}{x_3}$$

are unsolvable.

There are a number of related questions which are still open and require further study. Some obvious examples are:

1. Can the bound on $\lambda(k; a_1, a_2, a_3)$ be improved?

2. Can similar bounds be obtained for $\lambda(k; a_1, a_2, a_3, a_4)$ or more generally for $\lambda(k; a_1, \ldots, a_r)$? (One result along these lines is that $k = o(\lambda(k))$. This is obvious from Lemma 1 of [6].)

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