too. Therefore $A \cup B \neq \emptyset$. If $A = \emptyset$, then any line bu_0 where $b \in B$ is superfluous as can be verified (cf. Fig. 9). If $B = \emptyset$, then any line $u_1 a$ where $a \in A$ is superfluous. Thus $A \neq \emptyset$, $B \neq \emptyset$, and $C \neq \emptyset$. Then, however, any line ba, where $a \in A$ and $b \in B$, appears to be superfluous. Hence the case $m = 1$ is impossible.

(II) $m \geqq 2$.

As the path $u_0 u_1 \ldots u_m$ is a shortest $w_1 - w_2$ path, there is no line $u_i u_j$ whenever $j - i \geqq 2$. According to (1), there is no line $u_i b$ with $i \leqq k - 1$, $b \in B$, and no line au_j with $a \in A$, $j \geqq k + 2$. These facts are illustrated by Fig. 10. (In this figure, any full line has priority over a dashed line, e.g. if $k = 0$, then all lines $u_k a$ with $a \in A$ exist.) If $A = \emptyset$, then $d_{T-u_0 u_1}(u_0, u_1) = \infty$, i.e., by (2) and (3) we have $m = 1$ which contradicts our assumption. Therefore $A \neq \emptyset$. Analogously $B \neq \emptyset$ (for otherwise it would be $d_{T-u_{m-1}u_m}(u_{m-1}, u_m) = \infty$).

Now we assert that any line $z = b_0 a_0$, where $a_0 \in A$ and $b_0 \in B$, is superfluous. As $d_{T-z}(b, a) \leqq 2$ for any $a \in A$ and any $b \in B$, it is sufficient to verify that

(i) for any path of the form $b_0 a_0 v$, where $v \notin A$, there is a $b_0 - v$ path of length not exceeding 2 and not containing the line z. This is clear (cf. Fig. 10) except the case $v = u_{k+1}$ with $k + 1 = m - 1$. In this case, however, there is no line $u_k w$ with $w \in B$ (existence of such line would contradict (1)). Thus $b_0 u_k u_{k+1}$ is the required path.

(ii) for any path of the form $vb_0 a_0$, where $v \notin B$ there is a $v - a_0$ path of length not exceeding 2 and not containing the line z. This can be easily verified (cf. Fig. 10) except the case $v = u_1 = u_k$. In this case, however, there is no line wu_2 with $w \in A$ (see (1)). So we can take the path $u_1 u_2 a_0$.

Hence neither the case $m \geqq 2$ is possible and the theorem is proved.

Thus we have the full characterization of all e-critical tournaments. Nevertheless, we have not succeeded in proving or disproving the existence of a v-critical tournament with diameter $d \geqq 3$. We conjecture that there exists an integer d_0 such that there is no v-critical tournament with diameter $d \geqq d_0$.

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