Let $p \in C_T^0$. *Then the equation*

$$-(-1)^k x^{(2k)}(t) + a_1 x^{(2k-1)}(t) + \ldots + a_1 x'(t) + f(x(t))\, x'(t) + g(x(t)) = p(t)$$

has at least one T-periodic solution if and only if

$$g(-\infty) < T^{-1} \int_0^T p(t)\, \mathrm{d}t < g(+\infty)\,. \tag{7}$$

Proof. Necessity follows at once from Lemma 4.1. Now if (7) holds, the function

$$s \mapsto g(s) - T^{-1} \int_0^T p(t)\, \mathrm{d}t$$

satisfies assumptions (C) and (D) and the function

$$F : s \mapsto \int_0^s \int_0^y f(u)\, \mathrm{d}u\, \mathrm{d}y$$

satisfies assumption (B). Thus, by Theorem 6.1, there exists at least one T-periodic solution of equation

$$-(-1)^k x^{(2k)}(t) + a_1 x^{(2k-1)}(t) + \ldots + a_{2k-1} x'(t) + f(x(t))\, x'(t) + \\ + g(x(t)) - T^{-1} \int_0^T p(t)\, \mathrm{d}t = p(t) - T^{-1} \int_0^T p(t)\, \mathrm{d}t\,,$$

and the proof is complete.

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