small, then for each $\xi \in [-2, 0]$ we have $\psi(\xi) < e^{2(\delta+1)} < \pi^2$, hence $0 < \sqrt{\psi(a)} < \pi$, and hence $\sin \sqrt{\psi(a)} > 0$. Now, easy verification that the sum of squares of the left-hand sides of the two above equalities equals to 1 completes the proof of the lemma.

Theorem 2. *Theorem 1 does not hold with $1/e$ replaced by any greater constant.*

Proof. Let $c > 1/e$. In virtue of Lemma there is d with $1/e < d < c$ such that $x(t) = ke^{at} \cos(bt)$, where $a < 0$, $\pi > b > 0$, is a solution of the equation

$$x'(t) = -dx(t-1), \quad x(-\pi/2b + 1) = 1.$$

The $0 < x(t) \leqq 1$ for $t \in (-\pi/2b, \pi/2b) = (u, v)$, and $x(t)$ is maximal in (u, v) for $t = u + 1$. Define a function g by $g(t) = -dx(t-1)$ for $t \in [u + 1, u + 2]$, $g(t) = -d$ for $t \in [u + 2, v]$, $g(t) = 0$ for $t \in (v, 3\pi/2b + 1]$, and let g be periodic with period $2\pi/b$. For every integer n put $u_n = 2\pi n/b + u$, $v_n = 2\pi n/b + v$. If $y_0(t) =$ constant for each $t \in [u_n, u_n + 1]$, and if y is the solution of the equation

$$y'(t) = g(t)\, y(t-1) \tag{7}$$

for $t > u_n + 1$ with y_0 as initial function then $y(t) = 0$ for each $t \geqq v_n$. However, every solution of (7) defined for all $t \in R$ is constant on each interval $[u_n, u_n + 1]$, consequently (7) has no non-trivial solution defined for all $t \in R$.

On the other hand, the equation (7) satisfies the assumptions of Theorem 1 with the constant $1/e$ replaced by c, since $\sup_t |g(t)| < c$, $\tau = 1$, and $\vartheta = 0$, q.e.d.

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