Proof. Let $\bar{\varphi}$ be the dispersion of $(\bar{q})$. It follows from Lemma 1 that there exists a constant $M > 0$ such that $\varphi(t) - t \leqq M$, $\bar{\varphi}(t) - t \leqq M$, $t \in [a, \infty)$.

Thus

$$\lim_{t\to\infty} \max_{x\in[t,\bar{\bar{\varphi}}(t)]} |\bar{q}'(x)| \, (\bar{\bar{\varphi}}(t) - t) = 0$$

where $\bar{\bar{\varphi}}(t) = \max(\varphi(t), \bar{\varphi}(t))$. This together with Lemma 3 implies the statement of the theorem.

Theorem 3. *Let* (q), $(\bar{q})$ *be oscillatory* $(t \to \infty)$ *differential equations such that* $q \in C^0[a, \infty)$, $q \in C^1[a, \infty)$, $\lim_{t\to\infty} (q(t) - \bar{q}(t)) = 0$, $\lim_{t\to\infty} q(t) = -\infty$, $|\bar{q}'(t)| \leqq$ const. *for* $t \in [a, \infty)$. *Let* φ *be the dispersion of* (q). *Then*

$$\lim_{t\to\infty} (\varphi(t) - t) = 0\,, \quad \lim_{t\to\infty} \varphi'(t) = 1\,, \quad \lim_{t\to\infty} \varphi''(t) = \lim_{t\to\infty} \varphi'''(t) = 0\,.$$

Proof. Let $C < 0$ be an arbitrary number. As $\lim_{t\to\infty} \bar{q}(t) = -\infty$, there exists a number t_1, $t_1 \in [a, \infty)$ such that $q(t) < C$, $t \in [t_1, \infty)$. From the Sturm Comparison Theorem for the equations (q) and $y'' = C \,.\, y$ we obtain

$$0 < \varphi(t) - t \leqq \frac{\pi}{\sqrt{-C}}\,, \quad t \in [t_1, \infty)\,.$$

Hence $\lim_{t\to\infty} (\varphi(t) - t) = 0$. We can prove similarly that $\lim_{t\to\infty} (\bar{\varphi}(t) - t) = 0$ where $\bar{\varphi}$ is the dispersion of $(\bar{q})$. Thus

$$\lim_{t\to\infty} \max_{x\in[t,\bar{\bar{\varphi}}(t)]} |\bar{q}'(x)| \, (\bar{\bar{\varphi}}(t) - t) = 0$$

where $\bar{\bar{\varphi}}(t) = \max(\varphi(t), \bar{\varphi}(t))$ and the statement of the theorem follows from Lemma 3.

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