

## Werk

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**Proof.** Let  $\bar{\varphi}$  be the dispersion of  $(\bar{q})$ . It follows from Lemma 1 that there exists a constant  $M > 0$  such that  $\varphi(t) - t \leq M$ ,  $\bar{\varphi}(t) - t \leq M$ ,  $t \in [a, \infty)$ .

Thus

$$\lim_{t \rightarrow \infty} \max_{x \in [t, \bar{\varphi}(t)]} |\bar{q}'(x)| (\bar{\varphi}(t) - t) = 0$$

where  $\bar{\varphi}(t) = \max(\varphi(t), \bar{\varphi}(t))$ . This together with Lemma 3 implies the statement of the theorem.

**Theorem 3.** Let  $(q), (\bar{q})$  be oscillatory ( $t \rightarrow \infty$ ) differential equations such that  $q \in C^0[a, \infty)$ ,  $q \in C^1[a, \infty)$ ,  $\lim_{t \rightarrow \infty} (q(t) - \bar{q}(t)) = 0$ ,  $\lim_{t \rightarrow \infty} q(t) = -\infty$ ,  $|\bar{q}'(t)| \leq \text{const.}$  for  $t \in [a, \infty)$ . Let  $\varphi$  be the dispersion of  $(q)$ . Then

$$\lim_{t \rightarrow \infty} (\varphi(t) - t) = 0, \quad \lim_{t \rightarrow \infty} \varphi'(t) = 1, \quad \lim_{t \rightarrow \infty} \varphi''(t) = \lim_{t \rightarrow \infty} \varphi'''(t) = 0.$$

**Proof.** Let  $C < 0$  be an arbitrary number. As  $\lim_{t \rightarrow \infty} \bar{q}(t) = -\infty$ , there exists a number  $t_1, t_1 \in [a, \infty)$  such that  $q(t) < C$ ,  $t \in [t_1, \infty)$ . From the Sturm Comparison Theorem for the equations  $(q)$  and  $y'' = C \cdot y$  we obtain

$$0 < \varphi(t) - t \leq \frac{\pi}{\sqrt{-C}}, \quad t \in [t_1, \infty).$$

Hence  $\lim_{t \rightarrow \infty} (\varphi(t) - t) = 0$ . We can prove similarly that  $\lim_{t \rightarrow \infty} (\bar{\varphi}(t) - t) = 0$  where  $\bar{\varphi}$  is the dispersion of  $(\bar{q})$ . Thus

$$\lim_{t \rightarrow \infty} \max_{x \in [t, \bar{\varphi}(t)]} |\bar{q}'(x)| (\bar{\varphi}(t) - t) = 0$$

where  $\bar{\varphi}(t) = \max(\varphi(t), \bar{\varphi}(t))$  and the statement of the theorem follows from Lemma 3.

#### References

- [1] Bartušek M.: On Asymptotic Properties and Distribution of Zeros of Solutions of  $y'' = q(t)y$ . Acta F.R.N. Univ. Comenian. To appear.
- [2] Borůvka O.: Lineare Differentialtransformationen 2. Ordnung. VEB Berlin 1967.
- [3] Staněk J.: Poznámka k asymptotickým vlastnostem disperse rovnice  $y'' = q(t)y$ . Unpublished. A lecture in the seminar of Matematický Ústav ČSAV Brno.

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