Proof. It is clear that it suffices to prove that the function $t \to \mathcal{T}(t, 0)\, x$ is almost periodic for $x \in B$. For any $t \in R$ the operator $\mathcal{T}(t, 0)$ has the form $\mathcal{T}(t, 0) = T(\bar{A}_0, t)\, T(\overline{C(t)}, 1)$. The function $t \to T(\bar{A}_0, t)\, x$ is almost periodic for $x \in B$ which implies that there exists a constant $k_1 < \infty$ such that $|T(\bar{A}_0, t)| \leqq k_1$ for $t \in R$. If we prove that the function $t \to T(\overline{C(t)}, 1)\, x$ is almost periodic for $x \in B$, then the almost periodicity of the function $t \to \mathcal{T}(t, 0)\, x$ for $x \in B$ will follow from the well known theorem about the almost periodicity of composed function. Since the operators $T(\overline{C(t)}, 1)$ are uniformly bounded ($|T(\overline{C(t)}, 1)| \leqq e^{\omega_2}$ for $t \in R$), it suffices to prove that the function $t \to T(\overline{C(t)}, 1)\, x$ is almost periodic for $x \in D(A)$.

Let $x \in D(A)$. Then we have for $t, s \in R$ $T(\overline{C(t)}, 1)\, x - T(\overline{C(s)}, 1)\, x = T(\overline{C(s)}, 1)\,.\, (T(\overline{C(t) - C(s)}, 1)\, x - x) = T(\overline{C(s)}, 1)\, (\int_0^1 T(\overline{C(t) - C(s)}, r)\, (C(t)\, x - C(s)\, x)\, \mathrm{d}r)$. This implies that there exists a constant $k_2 = k_2(\omega_2)$ such that $|T(\overline{C(t)}, 1)\, x - T(\overline{C(s)}, 1)\, x| \leqq k_2 |C(t)\, x - C(s)\, x|$. The almost periodicity of the function $t \to T(C(t), 1)\, x$ follows from the assumption 3 of the Theorem.

Note. Let $\bar{A}_0$ be such a G-operator in a weakly complete Banach space B that $T(\bar{A}_0)$ is a bounded group of operators, i.e. $|T(\bar{A}_0, t)| \leqq k < \infty$ for $t \in R$. Moreover, let $\sigma(\bar{A}_0) = C \setminus \varrho(\bar{A}_0)$ be at most countable. Then it follows from Theorem 1 of [3] and from Theorem 3 of [4] that the assumption 3 of Theorem 2 is fulfilled, i.e., the function $t \to T(\bar{A}_0, t)\, x$ is almost periodic for any $x \in B$.

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