can be proved similarly. Finally take $\{F_n - f_n\}_{n=1}^{\infty}$. Evidently $F_n - f_n \searrow 0$, hence

$$0 = \bigwedge \mathscr{I}_0(F_n - f_n) = \bigwedge_n (\mathscr{I}_0(F_n) - \mathscr{I}_0(f_n)) = \bigwedge_n \mathscr{I}_0(F_n) - \bigvee_n \mathscr{I}_0(f_n)$$

and the Lemma is proved.

Definition 5.1. Denote by $\mathscr{I}(f)$ the common value in Lemma 5.1.

Theorem 5.1. *$\mathscr{K}$ is a linear lattice and $\mathscr{I}$ is a linear and non-decreasing function on $\mathscr{K}$.*

Proof. The first part of the Theorem follows directly from Theorems 4.1 and 4.2, and the other part can be obtained by simple computation from the definition of $\mathscr{I}$ and the properties of $\mathscr{I}_0$.

Theorem 5.2. *If $f_n \in \mathscr{K}$ $(n = 1, 2, \ldots)$, $f_n \nearrow f$ $(f_n \searrow f)$ and f is bounded, then $f \in \mathscr{K}$ and $\mathscr{I}(f_n) \nearrow \mathscr{I}(f)$ $((\mathscr{I}(f_n) \searrow \mathscr{I}(f))$.*

Proof. According to Theorem 2.6 $f \in \mathscr{M}_{\nearrow}$, hence $f \in \mathscr{A} \cap \mathscr{M}_{\nearrow} = \mathscr{K}$. Now use Theorem 2.6. If g_n $(n = 1, 2, \ldots)$ are the functions constructed in its proof, then $g_n \leqq f_n$, $g_n \in \mathscr{K}_0$, $g_n \leqq g_{n+1}$ $(n = 1, 2, \ldots)$ and $g_n \nearrow f$, hence

$$\mathscr{I}(f) = \bigvee \mathscr{I}_0(g_n) \leqq \bigvee \mathscr{I}_0(f_n) \leqq \mathscr{I}(f) .$$

Theorem 5.3. *Let $g \in \mathscr{K}$, $f_n \in \mathscr{K}$, $|f_n| \leqq g$ $(n = 1, 2, \ldots)$ and $f_n \to f$* (i.e., $f_n(x) \to$ *$\to g(x)$ in the order topology of Y at each $x \in X$). Then $f \in \mathscr{K}$ and $\mathscr{I}(f_n) \to \mathscr{I}(f)$.*

Proof. Put $g_n = \bigvee_{i=n}^{\infty} f_i$, $h_n = \bigwedge_{i=n}^{\infty} f_i$. Then $-g \leqq h_n \leqq f_n \leqq g_n \leqq g$, $g_n, h_n \in \mathscr{K}$ $(n = 1, 2, \ldots)$. Moreover, $h_n \nearrow f$, $g_n \searrow f$, hence $f \in \mathscr{K}$ and

$$\mathscr{I}(f) = \bigvee_{n=1}^{\infty} \mathscr{I}(h_n) \leqq \bigvee_{n=1}^{\infty} \bigwedge_{i=n}^{\infty} \mathscr{I}(f_i) = \liminf_{n\to\infty} \mathscr{I}(f_n) \leqq \limsup_{n\to\infty} \mathscr{I}(f_n) =$$

$$= \bigwedge_{n=1}^{\infty} \bigvee_{i=n}^{\infty} \mathscr{I}(f_i) \leqq \bigwedge_{n=1}^{\infty} \mathscr{I}(g_n) = \mathscr{I}(f) .$$

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Authors' address: 816 31 Bratislava, Mlynská dolina (Prírodovedecká fakulta UK).