

## Werk

**Label:** Table of literature references

**Jahr:** 1975

**PURL:** [https://resolver.sub.uni-goettingen.de/purl?31311157X\\_0100|log14](https://resolver.sub.uni-goettingen.de/purl?31311157X_0100|log14)

## Kontakt/Contact

[Digizeitschriften e.V.](#)  
SUB Göttingen  
Platz der Göttinger Sieben 1  
37073 Göttingen

✉ [info@digizeitschriften.de](mailto:info@digizeitschriften.de)

can be proved similarly. Finally take  $\{F_n - f_n\}_{n=1}^\infty$ . Evidently  $F_n - f_n \searrow 0$ , hence

$$0 = \bigwedge_n \mathcal{I}_0(F_n - f_n) = \bigwedge_n (\mathcal{I}_0(F_n) - \mathcal{I}_0(f_n)) = \bigwedge_n \mathcal{I}_0(F_n) - \bigvee_n \mathcal{I}_0(f_n)$$

and the Lemma is proved.

**Definition 5.1.** Denote by  $\mathcal{I}(f)$  the common value in Lemma 5.1.

**Theorem 5.1.**  $\mathcal{K}$  is a linear lattice and  $\mathcal{I}$  is a linear and non-decreasing function on  $\mathcal{K}$ .

*Proof.* The first part of the Theorem follows directly from Theorems 4.1 and 4.2, and the other part can be obtained by simple computation from the definition of  $\mathcal{I}$  and the properties of  $\mathcal{I}_0$ .

**Theorem 5.2.** If  $f_n \in \mathcal{K}$  ( $n = 1, 2, \dots$ ),  $f_n \nearrow f$  ( $f_n \searrow f$ ) and  $f$  is bounded, then  $f \in \mathcal{K}$  and  $\mathcal{I}(f_n) \nearrow \mathcal{I}(f)$  ( $\mathcal{I}(f_n) \searrow \mathcal{I}(f)$ ).

*Proof.* According to Theorem 2.6  $f \in \mathcal{M}_\nearrow$ , hence  $f \in \mathcal{A} \cap \mathcal{M}_\nearrow = \mathcal{K}$ . Now use Theorem 2.6. If  $g_n$  ( $n = 1, 2, \dots$ ) are the functions constructed in its proof, then  $g_n \leq f_n$ ,  $g_n \in \mathcal{K}_0$ ,  $g_n \leq g_{n+1}$  ( $n = 1, 2, \dots$ ) and  $g_n \nearrow f$ , hence

$$\mathcal{I}(f) = \bigvee_n \mathcal{I}_0(g_n) \leq \bigvee_n \mathcal{I}_0(f_n) \leq \mathcal{I}(f).$$

**Theorem 5.3.** Let  $g \in \mathcal{K}$ ,  $f_n \in \mathcal{K}$ ,  $|f_n| \leq g$  ( $n = 1, 2, \dots$ ) and  $f_n \rightarrow f$  (i.e.,  $f_n(x) \rightarrow g(x)$  in the order topology of  $Y$  at each  $x \in X$ ). Then  $f \in \mathcal{K}$  and  $\mathcal{I}(f_n) \rightarrow \mathcal{I}(f)$ .

*Proof.* Put  $g_n = \bigvee_{i=n}^\infty f_i$ ,  $h_n = \bigwedge_{i=n}^\infty f_i$ . Then  $-g \leq h_n \leq f_n \leq g_n \leq g$ ,  $g_n, h_n \in \mathcal{K}$  ( $n = 1, 2, \dots$ ). Moreover,  $h_n \nearrow f$ ,  $g_n \searrow f$ , hence  $f \in \mathcal{K}$  and

$$\begin{aligned} \mathcal{I}(f) &= \bigvee_{n=1}^\infty \mathcal{I}(h_n) \leq \bigvee_{n=1}^\infty \bigwedge_{i=n}^\infty \mathcal{I}(f_i) = \liminf_{n \rightarrow \infty} \mathcal{I}(f_n) \leq \limsup_{n \rightarrow \infty} \mathcal{I}(f_n) = \\ &= \bigwedge_{n=1}^\infty \bigvee_{i=n}^\infty \mathcal{I}(f_i) \leq \bigwedge_{n=1}^\infty \mathcal{I}(g_n) = \mathcal{I}(f). \end{aligned}$$

#### References

- [1] Dravecký J., Neubrunn T.: Measurability of functions of two variables, Mat. časop. SAV, 23 (1973), 147–157.
- [2] Sikorski R.: Funkcje rzeczywiste, Warszawa 1958.
- [3] Riečan B.: A note on measurable functions, Čas. pěst. mat. 96 (1971), 67–72.

*Authors' address:* 816 31 Bratislava, Mlynská dolina (Prírodovedecká fakulta UK).