Let $f_n \searrow 0$, f_1 be simple. Put $M = \max f_1$. Let $f_1 = \sum_{i=1}^{r} c_i \chi_{F_i}$. Take ε such that

$$\varepsilon \sum_{i=1}^{r} |F_i| < 2^{-m-1} .$$

Further put $E_n = \{x; f_n(x) \geqq \varepsilon\}$. Then $E_n \supset E_{n+1}$ $(n = 1, 2, \ldots)$, $\bigcap_{n=1}^{\infty} E_n = \emptyset$. Since $E_n \subset \bigcup_{i=1}^{r} F_i$ and f_1 is simple, $E_n \in \mathcal{N}_0$ for all n. Choose k such that $2^k > 2^{m+1} M$. Then there is n such that $E_n \in \mathcal{N}_k$. We get

$$f_n = f_n \chi_{F - E_n} + f_n \chi_{E_n} \leqq \varepsilon \chi_F + M \chi_{E_n} \leqq \varepsilon \sum_{i=1}^{r} \chi_{F_i} + M \chi_{E_n} .$$

Put $g = \varepsilon \sum_{i=1}^{r} \chi_{F_i} + M \chi_{E_n}$. Then

$$\sum_{i=1}^{r} \varepsilon |F_i| + M |E_n| \leqq 2^{-n-1} + M \cdot 2^{-k} < 2^{-m} ,$$

hence $g \in \mathcal{F}_m$ and therefore $f_n \in \mathcal{F}_m$. Hence to any m there is n such that $f_n \in \mathcal{F}_m$. The condition iii is proved.

If $E \in \mathcal{N}_n$, then $|E| \leqq 2^{-n}$, hence $\chi_E \in \overline{\mathcal{F}}_n \subset \mathcal{F}_n$.

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Author's address: 816 31 Bratislava, Mlynská dolina (Prírodovedecká fakulta UK).