Theorem 2. *Let γ be an infinite regular cardinal. Then there does not exist a free complete (γ, ∞)-distributive vector lattice on γ complete generators.*

Proof. Suppose that X_0 is a complete (γ, ∞)-distributive vector lattice with a set A_0 of free complete generators, card $A_0 = \gamma$. Let $\mathfrak{m}$ be a cardinal, $\mathfrak{m} >$ card X_0. Let $B^0_{\mathfrak{m}} = B$ be as in Thm. C'. Now we use a similar method as in the proof of Thm. 1. Let X be as in Thm. B. We may put $B = B(e)$. Choose two distinct elements $a_0, a_1 \in A_0$ and denote $A_1 = A_0 \setminus \{a_0, a_1\}$. Then there exists a mapping f_1 of A_1 onto A and let f be a mapping of A_0 into X such that $f(a_0) = 0, f(a_1) = e$ and $f(a) = f_1(a)$ for each $a \in A_0$.

Let Y be the closed vector sublattice of X generated by the set $A \cup \{0, e\}$. Then Y is a complete vector lattice that is completely generated by the set $A \cup \{e\}$ and e is a weak unit of Y. Let Y_0 be the set of all $y \in Y$ satisfying $-n(y)\, e \leqq y \leqq n(y)\, e$ for a positive integer $n(y)$. The set Y_0 is a complete vector lattice and it is a convex vector sublattice of Y; the element e is a strong unit of Y_0.

Let M be the Stone space of the Boolean algebra B. According to Thm. D, $C(M)$ is (γ, ∞)-distributive and hence by the Lemma the vector lattice $B(M)$ is (γ, ∞)-distributive. From Thm. E it follows that Y_0 is isomorphic with $B(M)$ and therefore Y_0 is (γ, ∞)-distributive. Since e is a weak unit of Y and since e belongs to Y_0, according to the Lemma we obtain that Y is (γ, ∞)-distributive. Thus there is a complete homomorphism ψ of X_0 onto Y. By the same reasoning as in the proof of Thm. 1 we get that $B(e) \subset Y$. Therefore $\mathfrak{m} \leqq$ card $Y \leqq$ card X_0, which is a contradiction.

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