estimators of p_{ij}, $p_{i.}$, $p_{.j}$ for $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, s$, where n_{ij} denotes the number of observations $(x_t, y_t) \in (X_i \times Y_j)$ and $n_{i.} = \sum_{j=1}^{s} n_{ij}$, $n_{.j} = \sum_{i=1}^{r} n_{ij}$. Then $\hat{\delta}_\alpha^{(1)}(\xi, \eta) = 1 + \hat{\text{I}}_\alpha^{(1)}(\xi, \eta)$, $\hat{\delta}_\alpha^{(2)}(\xi, \eta) = 1 + \hat{\text{I}}_\alpha^{(2)}(\xi, \eta)$, where $\hat{\text{I}}_\alpha^{(1)}(\xi, \eta) = -\sum_{i=1}^{r} \sum_{j=1}^{s} \hat{p}_{ij}^\alpha \cdot$ $\cdot (p_{i.}\, p_{.j})^{1-\alpha}$ and $\hat{\text{I}}_\alpha^{(2)}(\xi, \eta) = -\sum_{i=1}^{r} \sum_{j=1}^{s} \hat{p}_{ij}^\alpha (\hat{p}_{i.}\, \hat{p}_{.j})^{1-\alpha}$, $\alpha \in (0, 1)$.

Theorem 6.* *Under the hypothesis* $\text{H}_0 : \text{P}_{\xi\eta} = \text{P}_\xi \times \text{P}_\eta$ *the statistic*

$$\text{Z}_n^{(1)} = \frac{2n}{\alpha(\alpha - 1)} \ln\left[1 - \hat{\delta}_\alpha^{(1)}(\xi, \eta)\right] \tag{24}$$

is asymptotically χ^2*-distributed with* $(rs - 1)$ *degrees of freedom and the statistic*

$$\text{Z}_n^{(2)} = \frac{2n}{\alpha(\alpha - 1)} \ln\left[1 - \hat{\delta}_\alpha^{(2)}(\xi, \eta)\right] \tag{25}$$

is asymptoticaly χ^2*-distributed with* $(r - 1)(s - 1)$ *degrees of freedom.*

Proof. The asymptotic distribution of $\text{Z}_n^{(1)}$ follows directly from the results in [17]. The asymptotic distribution of $\text{Z}_n^{(2)}$ is found by expanding $\text{Z}_n^{(2)}$ in the Taylor series retaining the terms of the second order at the point $p_{ij} = p_{i.}\, p_{.j}$, $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, s$. After arranging suitably the terms in the Taylor expansion we obtain

$$\text{Z}_n^{(2)} = \sum_{i=1}^{r} \sum_{j=1}^{s} \frac{(n_{ij} - np_{ij})^2}{np_{ij}} - \sum_{i=1}^{r} \frac{(n_{i.} - np_{i.})^2}{np_{i.}} - \sum_{j=1}^{s} \frac{(n_{.j} - np_{.j})^2}{np_{.j}} + nU_n =$$

$$= \sum_{i=1}^{r} \sum_{j=1}^{s} \frac{(n_{ij} - n_{i.}\, p_{.j} - n_{.j} p_{i.} - np_{i.} p_{.j})^2}{np_{i.}\, p_{.j}} + nU_n\,,$$

where $nU_n \to 0$ in probability. Then according to 3b.4.(iv) in [14], $\text{Z}_n^{(2)}$ is asymptotically χ^2-distributed with $(r - 1)(s - 1)$ degrees of freedom.

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*) The asymptotic distribution of $\hat{\delta}_\alpha^{(2)}(\xi, \eta)$ has been derived more generally in the author's work [22].

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