ON CONNECTED GRAPHS CONTAINING EXACTLY TWO POINTS OF THE SAME DEGREE

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Following BEHZAD and CHARTRAND [1], we shall say that a graph G with $p \geqq 2$ points is quasiperfect if it contains exactly two points v and w of the same degree. The points v and w will be called the exceptional points of G. (For basic notions of graph theory, see HARARY [2].)

By D_2 we shall denote a line. If p is an integer and $p \geqq 3$, then by D_p we shall denote the complement of a graph obtained from D_{p-1} by adding an isolated point. As it immediately follows from Theorem 2 (and from its proof) in [1], for any integer $p \geqq 2$ it holds that: (i) G is a connected quasiperfect graph with p points if and only if G is isomorphic to D_p; (ii) G is a disconnected quasiperfect graph with p points if and only if G is isomorphic to the complement $\bar{D}_p$ of the graph D_p; (iii) each exceptional point of D_p has degree $[p/2]$. (If x is a real number, then $[x]$ is the greatest integer n such that $n \leqq x$; similarly, $\{x\} = -[-x]$.)

Let p be any integer such that $p \geqq 2$. We shall investigate properties of the graph D_p.

Proposition. *D_p has $[p/2] . \{p/2\}$ lines.*

Theorem 1. *Let t and u be points of D_p having degree d and e, respectively. Then t and u are adjacent if and only if $d + e \geqq p$.*

Proof. The case $p = 2$ is obvious. Assume that $p = n \geqq 3$ and that for $p = n - 1$ the theorem is proved. Let $d \leqq e$.

The case when $e = p - 1$ is obvious. Assume that $e \leqq p - 2$; then t and u lie in D_{p-1}. The points t and u are adjacent in D_p if and only if they are not adjacent in D_{p-1}. The points t and u are not adjacent in D_{p-1} if and only if $(p - 1 - d) + (p - 1 - e) < p - 1$. Hence the theorem follows.

Corollary 1. *Let i be an integer, $1 \leqq i \leqq [p/2]$. By t_i and u_i we denote points of D_p*