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Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

info@digizeitschriften.de

4. *A arises by permutations of rows and columns from an A' such that for each matrix from P(A') the product of its proper minor by the complementary minor is non-zero if and only if these minors are principal.*

Proof. $1 \to 4$. A is pseudo-triangular according to the above combinatorial characterization. The triangular matrix A' with non-zero diagonal elements has the property 4.

$4 \to 3$, $3 \to 2$. Obvious.

$2 \to 1$. The Laplace expansion yields

$$\det B = \pm \det B_{RC} \det B_{N-R,N-C} \neq 0 \quad \text{for each} \quad B \in P(A).$$

In [1], K. Čulík has conjectured that algebraic characterizations of the above type of absolutely non-singular matrices remain true even when the conditions concerning all the matrices from $P(A)$ $(P(A'))$ are restricted to the matrix $A(A')$ only. This is confirmed in the following special case.

Let $r < n$ be positive integers and A be a hermitian positive semi-definite (complex-valued) $n \times n$ matrix. Suppose that for each $R \subset N$, $C \subset N$, $|R| = |C| = r$ it holds $\det A_{RC} \det A_{N-R,N-C} \neq 0$ *if and only if $R = C$. Then A is diagonal.*

Proof. In the Hadamard inequality

$$\det A_{RR} \det A_{N-R,N-R} \geqq \det A,$$

equality is attained for each $R \subset N$, $|R| = r$. Accordingly, the matrices $A_{R,N-R}$, $A_{N-R,R}$ are zero for each such R (v. [4]). Hence the off-diagonal elements of A are zero.

References

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Author's address: 115 67 Praha 1, Žitná 25, (Matematický ústav ČSAV).