where $M_l = \bigcup_{\tau_j \in H_l} [\alpha_{j-1}, \alpha_j]$. Let us mention that $M_l \cap N_l = \emptyset$ since $M_l \subset \bigcup_{\tau_j \in H_l} [\tau_j - \delta(\tau_j), \tau_j + \delta(\tau_j)]$ and $[\tau_j - \delta(\tau_j), \tau_j + \delta(\tau_j)] \cap N_l = \emptyset$ for any $\tau_j \in H_l$. Hence $N \cap M_l \subset N - N_l$ and we have

$$S_l \leqq (l + 1) \sum_{t \in N - N_l} 2|g(t)| < (l + 1)\, \varepsilon \,.\, (l + 1)^{-1} \,.\, 2^{-l} = \varepsilon \,.\, 2^{-l}$$

Therefore we have

$$|K(A)| < \varepsilon\left(\sum_{m=1}^{\infty} 2^{-m} + \sum_{l=0}^{\infty} 2^{-l}\right) = 3\varepsilon$$

and the proposition follows immediately from Def. 1.

Proof of Theorem 1. Let us define the set

$$N_S = \{t \in (a, b);\, g(t+) = g(t-),\, g(t) \neq g(t-)\}$$

and the function $g_S(t) = 0$, $t \in [a, b] - N_S$, $g_S(t) = g(t)$ for $t \in N_S$. We put $g_R = g - g_S$.

Since $Y \int_a^b f \, \mathrm{d}g$ exists by assumption and the existence of $Y \int_a^b f \, \mathrm{d}g_S$ and also the equality $Y \int_a^b f \, \mathrm{d}g_S = 0$ follows from Proposition 1,1 in [4] the integral $Y \int_a^b f \, \mathrm{d}g_R$ exists. Using Theorem 3,1 form [4] we obtain that $K \int_a^b f \, \mathrm{d}g_R$ exists and Proposition 1 yields the existence of $K^* \int_a^b f \, \mathrm{d}g_R$ and the equality $K^* \int_a^b f \, \mathrm{d}g_R = K \int_a^b f \, \mathrm{d}g_R = Y \int_a^b f \, \mathrm{d}g_R$. By Prop. 2 we obtain the existence of $K^* \int_a^b f \, \mathrm{d}g_S$ and $K^* \int_a^b f \, \mathrm{d}g_S = 0$. Thus the integral $K^* \int_a^b f \, \mathrm{d}g$ exists and

$$K^* \int_a^b f \, \mathrm{d}g = K^* \int_a^b f \, \mathrm{d}g_S + K^* \int_a^b f \, \mathrm{d}g_R = Y \int_a^b f \, \mathrm{d}g_R = Y \int_a^b f \, \mathrm{d}g \,.$$

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