10. Důsledek. *Je-li* $A \in \mathscr{A}_1$, $S = I - A$, *pak* L_A *i* L_S *je uzavřený konvexní kužel.*
Toto tvrzení ihned plyne z vět 9, 6 a 2.

11. Označení. Buď $\mathfrak{A} = \{L_A : A \in \mathscr{A}_1\}$. Dále při $L_A \in \mathfrak{A}$ kladme $L_A^\perp = L_S$, kde $S = I - A$.

12. Věta. *Trojice* $(\mathfrak{A}, \subset, \perp)$ *je uspořádaná množina s ortogonalitou.*

Důkaz. a) Buď $L_A \in \mathfrak{A}$, $L_A^\perp = L_S$. Pak $L_S^\perp = L_A$ v důsledku platnosti věty 9.

b) Nechť $L_{A_1}, L_{A_2} \in \mathfrak{A}$, $L_{A_1} \subset L_{A_2}$. Buď $L_{A_1}^\perp = L_{S_1}$, $L_{A_2}^\perp = L_{S_2}$. Je-li $y \in L_{S_2}$, podle nerovnosti (6) pro všechna $z \in L_{A_2}$ platí $\mathrm{Re}\,(z - 0, y) \leqq 0$, což tedy platí též pro všechna $z \in L_{A_1}$. Jelikož navíc $y = 0 + y$ a $0 \in L_{A_1}$, podle důsledku 4 je $y \in L_{S_1}$. Tedy $L_{S_2} \subset L_{S_1}$.

c) Pro všechna $L_A \in \mathfrak{A}$ platí $L_A \subset L = L_I$, $L_A^\perp \subset L = L_I$. Nechť $L_A \subset L_B$, $L_A^\perp \subset L_B$, kde $L_B \in \mathfrak{A}$. Je-li $x \in L = L_I$, je $x = Ax + Sx$, kde $S = I - A$. Přitom $Ax \in L_A \subset L_B$ $Sx \in L_A^\perp \subset L_B$, tedy $x = Ax + Sx \in L_B$. Existuje proto $\sup(L_A, L_A^\perp)$ a platí $\sup(L_A, L_A^\perp) = L = L_I$.

Adresa autora: 166 27 Praha 6 - Dejvice, Suchbátarova 2 (katedra matematiky FEL ČVUT).

Summary

GENERALIZATION OF THE NOTION OF THE PROJECTOR

JAN HAVRDA, Praha

In the paper, the notion of a projector on a linear space L with the scalar product $(.,.)$ (and the induced norm $\|.\|$) is generalized. The set which is projected upon is a nonvoid closed convex set.

If $A : L \to L$ is such an idempotent operator that for all $x_1, x_2 \in L$ the inequality (1) holds, then $L_A = \{Ax : x \in L\}$ is a nonvoid convex closed set and for all $x_0 \in L$ and $z \in L_A$ the inequality (2) holds. Ax_0 is the only point in L_A to satisfy the inequality (2) for every $z \in L_A$. Furthermore, every $x \in L$ can be expressed uniquely by (4) so that for all $z \in L_A$ the inequality (5) holds. If $S = I - A$, where I is the identity operator, then $L_S = \{Sx : x \in L\}$ is a convex cone. If L_A is a compact set then $L_S = L$.

Let $\mathscr{A}_1$ stand for the family of all idempotent operators $A : L \to L$ such that for all $x_1, x_2 \in L$ the inequality (1) holds and for all $x \in L$ and $\lambda \geqq 0$ the formula $A\lambda x = \lambda Ax$ is satisfied. Then the implication $A \in \mathscr{A}_1 \Rightarrow S \in \mathscr{A}_1$ holds. In this case, L_A and L_S are closed convex cones. Let $\mathfrak{A}$ be the set $\{L_A : A \in \mathscr{A}_1\}$ and let us denote $L_A^\perp = L_S$. Then $(\mathfrak{A}, \subset, \perp)$ is a poset with orthogonality.