Hence the integral $K \int_a^b f \, dg_R = K \int_a^b f \, dg_c + K \int_a^b f \, dg_{Rb}$ exists; this statement is an easy consequence of Prop. 2,2.

We can summarize the above results for the case $N_S = \emptyset$ in the following

**Theorem 3,1.** *If $f : [a, b] \to R$, $g \to BV(a, b$ is such that $g(t+) = g(t-)$ for some $t \in (a, b)$ implies $g(t) = g(t-)$ and if $Y \int_a^b f \, dg$ exists, then also $K \int_a^b f \, dg$ exists and both integrals are equal.*

In the general case, i.e. if $N_S \neq \emptyset$ the existence of $Y \int_a^b f \, dg$ implies not necessarily the existence of $K \int_a^b f \, dg$. This fact is shown in Example 2,1.

If we suppose that $f$ is bounded on $N_S(|f(t)| \leqq M$ for $t \in N_S)$ then we define $\tilde{f}(t) = f(t)$ for $t \in N_S, \tilde{f}(t) = 0$ for $t \in [a, b] - N_S$. Hence $\tilde{f}$ is bounded and from Corollary 2,1 we obtain the existence of $K \int_a^b \tilde{f} \, dg_S$ while Prop. 2,2 guarantees the existence of $K \int_a^b f \, dg_S$. Corollary 2,1 gives moreover $K \int_a^b f \, dg_S = 0$ because $g_S(t) = g_S(t+) - g_S(t-) = 0$ for all $t \in [a, b]$. This yields the following

**Theorem 3,2.** *If $f : [a, b] \to R$, $|f(t)| \leqq M$ for $t \in N_S$, $g \in BV(a, b)$ and $Y \int_a^b f \, dg$ exists then $K \int_a^b f \, dg$ exists and both integrals are equal.*

**Remark 3,1.** Evidently, if the set $N_S$ is finite, then the boundedness of $f$ on $N_S$ can be omitted.

**Corollary 3,1.** *If $f, g \in BV(a, b)$ then $K \int_a^b f \, dg$ exists and equals $Y \int_a^b f \, dg$.*

Proof. This statement follows from Corollary 1,3 which states the same result for the Young integral, from the boundedness of $f$ and from Theorem 3,2.

Finally, we mention the known fact (see [1]), that if we set $[a, b] = [0, 1]$, $g(t) = t$, $f(t) = \sin(1/t) - (1/t)\cos(1/t)$, for $t \in (0, 1]$, $f(0) = 0$ then the Perron integral $P \int_0^1 f \, dg$ exists and by Theorem 2,1 also the integral $K \int_0^1 f \, dg$ exists. It is also known that for this choice of $f$ and $g$ the Riemann integral does not exist. Since $g(t) = t$ is continuous in $[0, 1]$ we obtain that $Y \int_0^1 f \, dg$ cannot exist (cf. Theorems 1,3, 1,4) and so we have an example of functions $g \in BV(a, b)$, $f : [a, b] \to R$ such that $K \int_a^b f \, dg$ exists but $Y \int_a^b f \, dg$ does not.

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