Summary

ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS WITH DEGENERATE COEFFICIENTS

JIŘÍ CERHA, Praha

An imbedding theorem for weighted Sobolev spaces is proved. Then it is used to prove an existence theorem for generalized partial differential equations with at one point degenerate coefficients.

Let $p \geqq 1$, k a positive integer, $\Omega \subset R_N$ a bounded domain satisfying (2). Let $W_{p,w}^{(k)}(\Omega)$ denote the weighted Sobolev space with the norm (1). Let b_i; $i = 0, \ldots, k - 1$; be some continuous positive non-decreasing functions on $(0, \infty)$, $\int_0^t b_i(r)^{-1} \, dr < \infty$ for $i = 1, \ldots, k - 1$. Let a, w be continuous nonnegative functions on $\bar{\Omega}$, positive for $x \neq 0$. Let $a(x) = a(r, S)$ be non-decreasing in r for all S, where $r = |x|$, $S = x/r$. Let

$$\int_0^t b_0(r)^{-1} \left(\int_{K_t - K_r} w(x)^{-p/p-1} \, dx \right) dr < \infty \quad \text{if} \quad p > 1 \, ,$$

$$w = 1 \quad \text{and} \quad \int_0^t b_0(r)^{-1} \, dr < \infty \quad \text{if} \quad p = 1 \, ,$$

where $K_r = \{x \in \Omega : 0 < x < r\}$, $r > 0$. Let $v(x) = a(x) \, b_0(x) \ldots b_{k-1}(x) \, w(x)$, $x \in \Omega$.

Then $W_{p,v}^{(k)}(\Omega) \subset L_{1,a}(\Omega)$ algebraically and topologically. Moreover, let

$$A = \sum_{m=0}^{k} (-1)^m \sum_{|i|=m} D^i \sum_{|j|=m} a_{i,j} D^j$$

be a partial differential operator with bounded measurable coefficients, f an element of the dual space $L_{1,a}^*(\Omega)$. We define the generalized solution of the equation $Au = f$ by means of the sesquilinear form (3). Let this form be $W_{p,v}^{(k)}(\Omega)$ – elliptic. Then there exists a solution.