Indeed,

$$4f(\lambda) = 4(B(\lambda x, y) - B(x, \lambda y)) = \varphi(\lambda x, y) + i\varphi(\lambda x, iy) - (\varphi(x, \lambda y) + i\varphi(x, i\lambda y)) =$$

$$= \varphi(\lambda x, y) + i\varphi(\lambda x, iy) - (\varphi(\lambda y, x) + i\varphi(i\lambda y, x)) =$$

$$= |\lambda|^2 \left[\varphi\left(x, \frac{y}{\lambda}\right) + i\varphi\left(x, i\frac{y}{\lambda}\right) - \left(\varphi\left(y, \frac{x}{\lambda}\right) + i\varphi\left(iy, \frac{x}{\lambda}\right)\right)\right] =$$

$$= |\lambda|^2 \left[4B\left(x, \frac{y}{\lambda}\right) - \left(\varphi\left(\frac{x}{\lambda}, y\right) + i\varphi\left(\frac{x}{\lambda}, iy\right)\right)\right] =$$

$$= |\lambda|^2 \left(4B\left(x, \frac{y}{\lambda}\right) - 4B\left(\frac{x}{\lambda}, y\right)\right) = -4|\lambda|^2 f\left(\frac{1}{\lambda}\right).$$

According to our lemma $f(\lambda) = f(i) \operatorname{Im} \lambda$. In particular, $f(t) = 0$ for real t so that $B(tx, y) = B(x, ty)$ for all real t. If $\lambda = it$, t real, we obtain

$$B(itx, y) - B(x, ity) = f(it) = tf(i) = t(B(ix, y) - B(x, iy))\,;$$

using 3° and 4°, this yields $i(B(tx, y) + B(x, ty)) = 2itB(x, y)$ whence $2iB(tx, y) = 2itB(x, y)$ which proves 5° and completes the proof.

References

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