(with rectilinear edges) joining the open vertical intervals $G = \{c\} \times (I \setminus E_c)$ and $H = \{d\} \times (I \setminus E_d)$. (If $G = H = \emptyset$ we include the whole of S in F; while if, for example, $H = \emptyset$ but $G \neq \emptyset$ then we include the whole of S except for the open triangle joining G to the point (d, z), where (c, z) is the mid-point of G.) It is easy to verify that the resulting set F is closed, and it clearly has the other required properties.

Theorem 2. *Let $f : I \to I$ be such that there is a partition $I = \bigcup_{n=1}^{\infty} A_n$ with each restriction $f \mid A_n$ continuous. Then given $\varepsilon > 0$ there exists a closed set $F \subseteq I \times I$ such that $F \cap \operatorname{Gr}(f) = \emptyset$ and $m(F_x) \geqq 1 - \varepsilon$ for all $x \in I$.*

Proof. Let $\sum \varepsilon_n$ be a convergent series of positive terms with sum less than ε. By Theorem 1 there exists for each n a closed set $F_n \subseteq I \times I$ such that $F_n \cap \operatorname{Gr}(f \mid A_n) = \emptyset$ and $m[(F_n)_x] \geqq 1 - \varepsilon_n$ for all $x \in I$. The set $F = \cap F_n$ has the required properties.

Theorem 3. *There exists a function $f : I \times I$ of Baire class 1 such that I cannot be partitioned into countably many sets A_n with each restriction $f \mid A_n$ continuous.*

Proof. In view of Theorem 2, it is sufficient for f to have the property that $F \cap \operatorname{Gr}(f) \neq \emptyset$ for every closed set $F \subseteq I \times I$ which satisfies $F_x \neq \emptyset$ for all $x \in I$. It is known [3] that there exists a function with G_δ graph having the stated property; this is not quite enough, but the example constructed explicitly in [4] is lower semicontinuous and therefore in the first Baire class.

Note added 13 January 1973. In a paper by L. Keldysh (Sur les fonctions premières mesurables B, Dokl. Akd. Nauk SSSR (N.S.) 5 (1934), 192–197) it was shown that for every α there exists a function $f : I \to I$ of Baire class α, such that I cannot be partitioned into countably many sets A_n with each restriction f/A_n of class less than α, thereby answering a question of P. S. Novikov, who had already proved the result stated above as Theorem 3.

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