(A): *If* $\mathscr{S}(3)$ *is a void set, the assertion* $[\mathscr{A}2]$ *holds.*

(B): *If the set* $\mathscr{S}(3)$ *is non-void, the assertion* $[\mathscr{A}1]$ *is valid, where* $\mathscr{D}_1 = \mathscr{D}_2 = \mathfrak{h}^1_{\mathscr{S}(3)}$, $V = V_7$, $W = W_7$ *and* $R = R_7$.

3.4. Problem $(\mathscr{P}_3)$. **Theorem 3.4.1.** *Let the problem* $(\mathscr{P}_3)$ *with* $\alpha = 0$ *and* $\omega = p/q$ *be given, where p, q are relatively prime natural numbers.*

(A): *If p is an odd number, the assertion* $[\mathscr{A}2]$ *holds, where* $\mathscr{F} = \mathscr{C}(\mathscr{I}; \mathscr{H}^2_{2\pi})$ *or* $\mathscr{F} = \mathscr{C}^{(1)}(\mathscr{I}; \mathscr{H}^1_{2\pi})$, $\mathscr{F}_0 = \mathscr{H}^3_{2\pi}$ *and* $\mathscr{F}_1 = \mathscr{H}^2_{2\pi}$.

(B): *Let* $p = 2m$ *be an even number and let* $\mathscr{S}(4)$ *denote the set* $\{k \in \mathscr{M} \mid k/m = \text{odd number}\}$. *Then the assertion* $[\mathscr{A}1]$ *is valid, where* $\mathscr{F}$, $\mathscr{F}_0$ *and* $\mathscr{F}_1$ *are the same spaces as above and* $\mathscr{D}_1 = \mathfrak{h}^2_{\mathscr{S}(4)}$, $\mathscr{D}_2 = \mathscr{H}^2_{2\pi}$, $V = V_9$, $W = W_9$ *and* $R = R_9$.

Theorem 3.4.2. *Let the problem* $(\mathscr{P}_3)$ *with* $\alpha = 0$ *be given, where the number* ω *satisfies the assumption* $[\mathscr{L}\varrho]$ *for a natural* $\varrho \geqq 2$. *Then the assertion* $[\mathscr{A}2]$ *is valid, where* $\mathscr{F} = \mathscr{C}(\mathscr{I}; \mathscr{H}^{\varrho+1}_{2\pi})$ *or* $\mathscr{F} = \mathscr{C}^{(1)}(\mathscr{I}; \mathscr{H}^{\varrho}_{2\pi})$, $\mathscr{F}_0 = \mathscr{H}^{\varrho+2}_{2\pi}$ *and* $\mathscr{F}_1 = \mathscr{H}^{\varrho+1}_{2\pi}$.

Theorem 3.4.3. *Let the problem* $(\mathscr{P}_3)$ *with* $\alpha \neq 0$ *and* $\omega = p/q$ *be given, where p, q are relatively prime natural numbers. Let* $\mathscr{S}(5)$ *be the set* $\{k \in \mathscr{M} \mid S_5(k) = 0\}$, *where* $S_5(k)$ *is defined by* (2.4.14), *and let the spaces* $\mathscr{F}$, $\mathscr{F}_0$, $\mathscr{F}_1$ *have one of the following meanings:*

(*i*) *If p is an odd number,* $\mathscr{F} = \mathscr{C}(\mathscr{I}; \mathscr{H}^2_{2\pi})$ *or* $\mathscr{F} = \mathscr{C}^{(1)}(\mathscr{I}; \mathscr{H}^1_{2\pi})$, $\mathscr{F}_0 = \mathscr{H}^3_{2\pi}$ *and* $\mathscr{F}_1 = \mathscr{H}^2_{2\pi}$.

(*ii*) *If* $p = 2m$ *is an even number, either* $\mathscr{F} = \mathscr{C}(\mathscr{I}; \mathscr{H}^3_{2\pi})$ (*or* $\mathscr{F} = \mathscr{C}^{(1)}(\mathscr{I}; \mathscr{H}^2_{2\pi})$), $\mathscr{F}_0 = \mathscr{H}^4_{2\pi}$, $\mathscr{F}_1 = \mathscr{H}^3_{2\pi}$ *or* $\mathscr{F} = \mathscr{C}(\mathscr{I}; [\mathscr{H}^2_{2\pi}]^{\perp}_{\mathscr{S}(4)})$ (*or* $\mathscr{F} = \mathscr{C}^{(1)}(\mathscr{I}; [\mathscr{H}^1_{2\pi}]^{\perp}_{\mathscr{S}(4)})$), $\mathscr{F}_0 = [\mathscr{H}^3_{2\pi}]^{\perp}_{\mathscr{S}(4)}$, $\mathscr{F}_1 = [\mathscr{H}^2_{2\pi}]^{\perp}_{\mathscr{S}(4)}$, *where* $\mathscr{S}(4)$ *means the set* $\{k \in \mathscr{M} \mid k/m = \text{odd number}\}$.

Then the following propositions hold:

(A): *If* $\mathscr{S}(5)$ *is a void set, the assertion* $[\mathscr{A}2]$ *is valid.*

(B): *If the set* $\mathscr{S}(5)$ *is non-void, the assertion* $[\mathscr{A}1]$ *holds, where* $\mathscr{D}_1 = \mathscr{D}_2 = \mathfrak{h}^1_{\mathscr{S}(5)}$, $V = V_{10}$, $W = W_{10}$ *and* $R = R_{10}$.

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