In this example, formula (6) was used to evaluate $d(P)$. However, for a computer the recursive formula (5) or (7) is much more suitable.

Problem 2. Given an arbitrary matrix $M = \|x_{ij}\|$ of the type m/n, i.e. with $m \geqq 1$ rows and $n \geqq 1$ columns, then the sequence of its rows $R_M = (\varrho_1, \varrho_2, \ldots, \varrho_m)$ where $\varrho_p = (x_{p1}, x_{p2}, \ldots, x_{pn})$ for $p = 1, 2, \ldots, m$ and the sequence of its columns $S_M = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ where $\sigma_q = (x_{1q}, x_{2q}, \ldots, x_{mq})$ for q $= 1, 2, \ldots, n$ determine two numbers which characterize the matrix M, namely, the numbers $a(R_M)$ and $a(S_M)$ which we shall call the *row-* and *column characterstics* of the matrix M. Obviously $n \leqq a(R_M) \leqq n \,.\, m$ and $m \leqq a(S_M) \leqq n \,.\, m$ for any matrix M of the type m/n.

Both sequences R_M and S_M are very closely related and therefore it may be expected that the numbers $a(R_M)$ and $a(S_M)$ will depend on each other in some manner. For example, it is apparent that these numbers assume their minimum value simultaneously, namely, when $x_{ij} = x_{hk}$ for all i, j, h, k. Of what kind is the relation between the two numbers?

When evaluating the numbers $a(R_M)$ and $a(S_M)$ according to formula (1), the repeating of elements x_{hj} and x_{ij} (in the same places) is examined. Such a repeating shows a dependence between the strings and hence the ranks of some submatrices depend on it. Is it possible to find some relation between the row- and column characteristics of a matrix and its rank or the ranks of its submatrices?

If the elements of the matrix x_{ij} are arbitrary numbers, then for $a(R_M)$ to assume its maximum value it is necessary and sufficient that $x_{i1} \neq x_{h1}$ for all $i \neq h$, where $i, h = 1, 2, \ldots, m$. By interchanging the columns of the matrix, this number cannot decrease below $m + n - 1$. If M is an incidence matrix (its elements x_{ij} being either 1 or 0) of the type m/n, where $m, n > 2$, then $a(R_M)$ cannot assume its theoretical maximum value $m \,.\, n$. The question arises, what is the maximum possible value with respect to all the possible interchanges of the columns. An analogous question may be formulated for the rows.

The propounded problem of the maximum value of the number $a(R_M)$ with respect to the interchanges of columns may be generalized to the problem of its maximum value under the assumption that M has a prescribed number of ones (other elements being zero).

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