of U_{n-1}. As T is a tree, any vertex of U_n is joined by an edge exactly with one vertex of U_{n-1}. Thus if $v \in U_{n-1}$, then let $V(v)$ be the set of vertices of U_n joined with v. We have $V(v) \wedge V(w) = \emptyset$ for $v \neq w$, $v \in U_{n-1}$, $w \in U_{n-1}$. The cardinality of $V(v)$ is at most $\aleph_\alpha$. We can well-order the set $V(v)$ so that the corresponding ordinal number is at most ω_α. Thus we have a transfinite sequence $\{w_\iota\}_{\iota<\beta}$ where $\beta \leqq \omega_\alpha$, of all elements of $V(v)$. Let the matrix assigned to v be $\|m_{i\varkappa}\|$; as $v \in U_{n-1}$, we assume that $m_{i\varkappa} = 0$ for $i > n - 1$. To any vertex w_μ of this sequence we assign the matrix $\|m'_{i\varkappa}\| \in \mathfrak{M}_0(\omega_\alpha, \omega_0)$ such that $m'_{n\mu} = 1$, $m'_{n\varkappa} = 0$ for $\varkappa \neq \mu$, $m'_{i\varkappa} = m_{i\varkappa}$ for $i \neq n$.

The matrix assigned to a vertex of U_n for $n > 0$ has some 1 in the n-th row and only zeroes in the further rows; thus no matrix can be assigned to two vertices of different sets U_n. We shall prove by induction according to n that for any two different vertices of U_n the corresponding matrices are different. The set U_0 contains only one vertex, thus it fulfills this assertion trivially. In a set U_n for some $n > 0$ any vertex belongs to exactly one $V(v)$ for $v \in U_{n-1}$. The matrix assigned to a vertex of $V(v)$ has the first $n - 1$ rows equal to these rows in the matrix assigned to v. If $v \in U_{n-1}$, $w \in U_{n-1}$, $v \neq w$, the matrices (according to the induction assumption) assigned to v and w are different; they differ in some element of the first $n - 1$ rows (because the following rows consist only of zeroes). Thus if we take a vertex of $V(v)$ and a vertex of $V(w)$, the assigned matrices to these vertices must differ, too. Finally, if we have two different vertices of the same $V(v)$, the assigned matrices differ in the n-th row. We have proved that the assigning matrices of $\mathfrak{M}_0(\omega_\alpha, \omega_0)$ to the vertices of T is one-to-one.

Further we see that for any pair of vertices joined by an edge the assigned matrices differ exactly in one element.

Now if $u \in U$ and $\boldsymbol{M}$ is the matrix assigned to u, we put

$$\mathfrak{v}(u) = \Phi(\boldsymbol{M}).$$

Thus $\mathfrak{v}(u)$ is some transfinite sequence of the ordinal ω_α and if u, v are two vertices of T joined by an edge, then $\mathfrak{v}(u)$ and $\mathfrak{v}(v)$ differ exactly in one element. If we identify any vertex u of T with the vertex of $Q(\aleph_\alpha)$ corresponding to the sequence $\mathfrak{v}(u)$, we have embedded T into $Q(\aleph_\alpha)$, q.e.d.

It is easy to prove that the cardinality of the vertex set of $Q(\aleph_\alpha)$ for any infinite cardinal number $\aleph_\alpha$ is equal to $\aleph_\alpha$. Thus no graph whose vertex set has the cardinality $\aleph_\alpha$ can be embedded into $Q(\aleph_\beta)$, where $\aleph_\beta < \aleph_\alpha$.

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