ii) Let us note that the existence of a linear Gâteaux differential $DF(u, h)$ of $F : X \to Y$ in some convex neighborhood $V(u_0)$ of $u_0 \in X$ and its joint continuity at (u_0, u_0) does not imply the existence of the Fréchet derivative $F'(u_0)$, in general. A. Alexiewicz and W. Orlicz [11] proved that there exists a mapping $f : c_0 \to c_0$ satisfying the condition of Lipschitz, having everywhere linear Gâteaux differential $DF(u, h)$ continuous in (u, h) jointly and being nowhere Fréchet differentiable. The above conclusion does not hold even if we impose on X more restrictive conditions. Indeed, let $h(u) = g(u(x), x)$ be an operator of Nemyckii, where a function $g(u, x)$ satisfies the conditions of Thm. 20.2 [5]. Then [12] $h(u)$ is Gâteaux – differentiable in the space L_2, $Dh(u, v)$ is continuous jointly in (u, v) on L_2 and $h(u)$ satisfies the condition of Lipschitz on L_2. However, $h(u)$ is nowhere Fréchet – differentiable in L_2.

iii) Let $F : X \to Y$ be a mapping having a bounded differential $DVF(u_0, h)$ at $u_0 \in X$. Then F is continuous at u_0. Indeed, $dVF(u_0, .)$ is bounded in some open neighborhood $U(0)$ of 0 and being homogeneous in $h \in X$, $dVF(u_0, .)$ is continuous at 0 by Thm. 2 (a). Hence for given $\varepsilon = 1$ there exists $\delta_0 > 0$ so that $h \in X$, $\|h\| \leqq \delta_0 \Rightarrow \Rightarrow \|dVF(u_0, h)\| \leqq 1$. Let $h \in X$ be arbitrary, then

$$\left\|\frac{h\delta_0}{\|h\|}\right\| = \delta_0 \Rightarrow \left\|dVF\left(u_0, \frac{h\delta_0}{\|h\|}\right)\right\| \leqq 1 .$$

Hence $\|dVF(u_0, h)\| \leqq \delta_0^{-1}\|h\|$ for each $h \in X$. Let $(u_n) \in X$, $u_0 \in X$, $u_n \to u_0$. Then $u_n = u_0 + z_n$, $(z_n) \in X$, where $z_n \to 0$. By our hypothesis

$$\|F(u_0 + z_n) - F(u_0)\| \leqq \|dVF(u_0, z_n)\| + \|\omega(u_0, z_n)\| \leqq \delta_0^{-1}\|z_n\| + o(\|z_n\|) .$$

Hence $F(u_n) \to F(u_0)$.

iiii) If $F : X \to Y$ is p – positively homogeneous on X, $(p > 1)$ and $\sup_{\|u\|=1} \|F(u)\| < < +\infty$, then F possesses the Fréchet derivative $F'(0)$ at 0 and $F'(0) = 0$.

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