On the other hand we can easily show that

$$\mathcal{U}_{1-\varphi} \bigcap^{\infty}(\lambda \notin \sigma_T(x)) (\lambda I - T) X \subset \bigcap^{\infty}(\lambda \notin \sigma_T(x)) (\lambda I - T) \mathcal{U}_{1-\varphi}X .$$

Take a $\lambda \in \sigma_T(x)$. By 2.5 the subspace $\mathcal{U}_{1-\varphi}X \subset X_T(\text{supp}\,(1 - \varphi))$ and thus the operator $(\lambda I - T)$ is one-to-one on $\mathcal{U}_{1-\varphi}X$. Let us show that $(\lambda I - T)$ maps $\mathcal{U}_{1-\varphi}X$ onto itself.

Let $x = \mathcal{U}_{1-\varphi}z$, then there is a $y \in X_T(\text{supp}\,(1 - \varphi))$ such that $x = (\lambda I - T)\, y$. Put $\psi(\lambda) = 0$ and $\psi(\mu) = (1 - \varphi(\mu))/(\lambda - a(\mu))$ for $\mu \neq \lambda$, so that $\psi \in C^\infty(R_2)$. Denote $u = \mathcal{U}_\psi z - y$. Since

$$(\lambda I - T)\, u = \mathcal{U}_{1-\varphi}z - (\lambda I - T)\, y = 0\,,$$

it is $\sigma_T(u) \subset \{\lambda\}$. On the other hand $\sigma_T(y) \subset \text{supp}\,(1 - \varphi)$, $\sigma_T(\mathcal{U}_\psi z) \subset \text{supp}\,(\psi) = \text{supp}\,(1 - \varphi)$ and thus $\sigma_T(u) \subset \text{supp}\,(1 - \varphi)$. But $\sigma_T(u) \subset \text{supp}\,(1 - \varphi) \cap \{\lambda\} = \emptyset$ and $u = 0$. We have obtained $y = \mathcal{U}_\psi z$. Let $\varphi_0 \in C^\infty(R_2)$, $\varphi_0 \equiv 1$ in a neighbourhood of $\sigma_T(y)$ such that $\text{supp}\,\varphi_0 \cap \sigma_T(x) = \emptyset$. Then $y = \mathcal{U}_{\varphi_0}y = \mathcal{U}_{1-\varphi}\mathcal{U}_{\varphi_0/(\lambda - a)}z \in \mathcal{U}_{1-\varphi}X$.

So we can write

$$\bigcap^{\infty}(\lambda \notin \sigma_T(x)) (\lambda I - T) \mathcal{U}_{1-\varphi}X = \bigcap^{\infty}(\lambda \in \mathbf{C}) (\lambda I - T) \mathcal{U}_{1-\varphi}X .$$

Since $\{0\}$ is the only T-divisible subspace, we have $\mathcal{U}_{1-\varphi}Sx = 0$ and $Sx = \mathcal{U}_{1-\varphi}Sx + \mathcal{U}_\varphi Sx = \mathcal{U}_\varphi Sx \in X_T(\text{supp}\,\varphi)$ for every $\varphi \in C^\infty(R_2)$ such that $\varphi \equiv 1$ in a neighbourhood of $\sigma_T(x)$. From this fact it follows obviously $\sigma_T(Sx) \subset \sigma_T(x)$.

Open problem: Is there a generalized scalar operator or a spectral operator of the finite type having a non-trivial divisible subspace? [1])

References

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[1]) The problem was solved by the author and the results will be published.