we have $f_k(x) \geqq 0$ and $\int_\Omega f_k(x)\,dx \to 0$. If necessary, for a subsequence, still noted f_k, $f_k(x) \to 0$ almost everywhere. This implies $(\partial u_k/\partial x_i)(x) \to (\partial u/\partial x_i)(x)$ almost everywhere. But (3.5) gives uniform continuity of the integrals $\int_M \sum_{i=1}^{n} |\partial u_k/\partial x_i|^m\,dx$ with respect to k. This implies $\partial u_k/\partial x_i \to \partial u/\partial x_i$ in $L_m(\Omega)$ for the original sequence.

Hence we obtain: the conditions (1.1), (3.3)–(3.5) being satisfied, there exists a solution $u \in W_m(\Omega)$ of (1.2), and every solution is such that

$$\|u\|_{W_m^{(1)}} \leqq c\big(1 + \sum_{i=0}^{n} \|f_i\|_{L_{m'}}^{1/(m-1)}\big) \tag{3.6}$$

if and only if $u = 0$

is the only solution of (1.2) for $f_i = 0$ and the coefficients A_i.

III) As far as the integral equation (1.4), although there is a lot of possible generalizations, we shall consider the condition:

$$\left|\frac{1}{t} f(y, tu) - a(y)\,u\right| \leqq c(t)\,(1 + |u|) \quad \text{with} \quad c(t) \to 0 \tag{3.7}$$

for $t \to \infty$ and $a \in L_\infty(M)$. The operators from $L_2(M) \to L_2(M)$ defined by

$$\int_M K(x, y)\, f(y, u(y))\,dy\,, \quad \int_M K(x, y)\, a(y)\, u(y)\,dy \tag{3.8}$$

are completely continuous. If we have

$$f(y, -u) = -f(y, u)\,, \tag{3.9}$$

we can immediately apply Theorem 2. But it is easy to see that we can immediately apply Theorem 1 in virtue of the complete continuity of (3.8) without (3.9). We obtain:

The equation (1.4) provided (3.7) has a solution for every $w \in L_2(M)$ and for every solution holds $\|u\|_{L_2} \leqq c(1 + \|w\|_{L_2})$ if and only if λ is not an eigenvalue for the linear equation

$$u(x) - \lambda \int_M K(x, y)\, a(y)\, u(y)\,dy = 0\,.$$

This result is very close to the corresponding result of M. A. Krasnoselskij [4].

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