Let $E \in \mathbf{S}(\mathbf{C})$ be an arbitrary set. Then $E = \bigcup_{n=1}^{\infty} E_n$, where $E_n \in \mathbf{S}(\mathbf{C})$, $E_n \subset E_{n+1}$, $E_n \subset C_n \in \mathbf{C}$ $(n = 1, 2, \ldots)$. Hence μ is inner $\mathbf{C}$-regular on $\mathbf{S}(\mathbf{C})$. By ([3], Theorem 1, p. 135) μ is $(\mathbf{C}, \mathbf{U})$-regular on $\mathbf{S}(\mathbf{D})$.

It is trivial that (e) $\Rightarrow$ (d) $\Rightarrow$ (b) and (e) $\Rightarrow$ (c) $\Rightarrow$ (b).

(b) $\Rightarrow$ (a): Since $\mathbf{C} \subset \mathbf{D}$, it is

$$\mu(U) = \sup \{\mu(D) : U \supset D \in \mathbf{D}\} \quad \text{for all} \quad U \in \mathbf{U}\,.$$

From the $(\mathbf{U}, \sigma)$-finiteness of μ it follows that $X = \bigcup_{n=1}^{\infty} U_n$, $U_n \in \mathbf{U}$, $\mu(U_n) < \infty$ $(n = 1, 2, \ldots)$. By ([3], Lemma 1, p. 136) there exist sets $Y_n \in \mathbf{S}(\mathbf{C})$ such that $\mu(U_n - Y_n) = 0$. Let $Y = \bigcup_{n=1}^{\infty} Y_n$. Then $Y \in \mathbf{S}(\mathbf{C})$ and $\mu(X - Y) \leqq \sum_{n=1}^{\infty} \mu(U_n - Y_n) = 0$.

Theorem 2. *If X is a locally compact Hausdorff space and μ is a $(\mathbf{U}, \sigma)$-finite measure on $\mathbf{S}(\mathbf{D})$, the conditions* (a)–(h) *are equivalent.*

Proof. It is trivial that (f) $\Rightarrow$ (g) $\Rightarrow$ (h).

(h) $\Rightarrow$ (e): From the $(\mathbf{U}, \sigma)$-finiteness of μ it follows that $\mu(C) < \infty$ for all $C \in \mathbf{C}$. If $C \in \mathbf{C}$ and $C \subset U \in \mathbf{U}$, there exists an open Baire set O such that $C \subset O \subset U$. Hence

$$\mu(C) = \inf \{\mu(U) : C \subset U,\ U \text{ open Baire set}\}\,.$$

This proves the $(\mathbf{C}, \mathbf{U})$-regularity of μ on $\mathbf{S}(\mathbf{C})$. By ([3], Theorem 1 p. 135) μ is $(\mathbf{C}, \mathbf{U})$-regular on $\mathbf{S}(\mathbf{D})$.

The other implications follow from Theorem 1.

Theorem 3. *If X is an arbitrary Hausdorff topological space and μ is a finite measure on $\mathbf{S}(\mathbf{D})$, the conditions* (a)–(h) *are equivalent.*

Proof. It is trivial that (f) $\Rightarrow$ (g) $\Rightarrow$ (h).

(h) $\Rightarrow$ (f): By ([2], Theorem 8, p. 43, or example 3 p. 45). The other implications follow from Theorem 1.

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