respectively, then (14) and (15) yield

$$[E^{-1}(\tfrac{1}{2}Be^{A})]^2 + [S^{-1}(B)]^2 = B^2 ,$$

from which we obtain (3).

This result is sharp because the relations (6) and (7) are sharp. This completes our proof.

We note that our result (4) is at variance with another recent result due to Dvořák [2; p. 180].

2. Dvořák also obtained the following result [2; p. 187].

Theorem C. *If $g(z) = z + a_3z^3 + \ldots$ is an analytic univalent odd function in the unit disc D, then*

$$Re(g(z)/z) > \tfrac{1}{2} \tag{16}$$

holds for $|z| < r_1'$, where r_1' is the smallest positive root of the equation

$$\sqrt{(r)}\log\frac{1+\sqrt{r}}{1-\sqrt{r}} = 2 .$$

A computation shows that

$$r_1' = 0{\cdot}913\ldots$$

We obtain the following *sharp* result.

Theorem D. *Let $g(z) = z + a_3z^3 + \ldots$ be analytic, univalent and odd in the unit disc D. Then the inequality (16) holds for $|z| < r_1$, where r_1 is the smallest positive root of the equation*

$$\left[S^{-1}\left(\tfrac{1}{2}\log\frac{1+\sqrt{r}}{1-\sqrt{r}}\right)\right]^2 + \left[E^{-1}\left(\tfrac{1}{2}\sqrt{(1-r)}\log\frac{1+\sqrt{r}}{1-\sqrt{r}}\right)\right]^2 = \left(\tfrac{1}{2}\log\frac{1+\sqrt{r}}{1-\sqrt{r}}\right)^2 .$$

This result is sharp. Moreover, a computation shows that

$$r_1 = 0{\cdot}914\ldots$$

Proof. If we get $f(z^2) = [g(z)]^2$, then $f(z)$ is analytic and univalent in the unit disc D. We then apply Theorem **B**. This completes the proof.

References

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