Example. It is well known that there exists such a trigonometrical series

$$\sum_{n=1}^{\infty} \varrho_n \cos(nx - \alpha_n) \quad \text{that} \quad \sum_{n=1}^{\infty} |\varrho_n \cos(nx - \alpha_n)| = +\infty \quad \text{a.e.}$$

and

$$\sum_{n=1}^{\infty} |\varrho_n \cos(nx - \alpha_n)| < +\infty$$

at every point of an uncountable set. We set

$$f(x) = (1 + \sum_{n=1}^{\infty} |\varrho_n \cos(nx - \alpha_n)|)^{-1} \quad \text{if} \quad \sum_{n=1}^{\infty} |\varrho_n \cos(nx - \alpha_n)| < +\infty\,,$$

$$f(x) = 0 \qquad \text{if} \quad \sum_{n=1}^{\infty} |\varrho_n \cos(nx - \alpha_n)| = +\infty\,.$$

If $f(x_0) = 0$, then f is continuous at x_0. If $f(x_0) > 0$, then we use the following inequalities

$$||\varrho_n \cos[n(x_0 + h) - \alpha_n]| - |\varrho_n \cos[n(x_0 - h) - \alpha_n]|| \leqq 2|\varrho_n \cos(nx_0 - \alpha_n)|$$

$$||\varrho_n \cos[n(x_0 + h) - \alpha_n]| - |\varrho_n \cos[n(x_0 - h) - \alpha_n]|| \leqq 2|\varrho_n|\,|\sin nh|\,.$$

From the first formula it follows that if $f(x_0 + h) = 0$ then $f(x_0 - h) = 0$. If $f(x_0 + h) > 0$ then for every N

$$|f(x_0 + h) - f(x_0 - h)| \leqq \sum_{n=1}^{\infty} ||\varrho_n \cos[n(x_0 + h) - \alpha_n]| - |\varrho_n \cos[n(x_0 - h) - \alpha_n]|| \leqq$$

$$\leqq \sum_{n=1}^{N} 2|\varrho_n|\,|\sin nh| + \sum_{n=N+1}^{\infty} |\varrho_n \cos(nx_0 - \alpha_n)|\,.$$

It follows easily that f is symmetrically continuous. Obviously f is not continuous at every point where it is positive.

References

[1] *H. Fried:* Über die symmetrische Stetigkeit von Funktionen, Fund. Math. *29* (1937), 134—137.
[2] *S. Saks:* Theory of the Integral, Warszawa 1937.

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