

Werk

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Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

✉ info@digizeitschriften.de

Example. It is well known that there exists such a trigonometrical series

$$\sum_{n=1}^{\infty} \varrho_n \cos (nx - \alpha_n) \quad \text{that} \quad \sum_{n=1}^{\infty} |\varrho_n \cos (nx - \alpha_n)| = +\infty \quad \text{a.e.}$$

and

$$\sum_{n=1}^{\infty} |\varrho_n \cos (nx - \alpha_n)| < +\infty$$

at every point of an uncountable set. We set

$$f(x) = \left(1 + \sum_{n=1}^{\infty} |\varrho_n \cos (nx - \alpha_n)|\right)^{-1} \quad \text{if} \quad \sum_{n=1}^{\infty} |\varrho_n \cos (nx - \alpha_n)| < +\infty,$$

$$f(x) = 0 \quad \text{if} \quad \sum_{n=1}^{\infty} |\varrho_n \cos (nx - \alpha_n)| = +\infty.$$

If $f(x_0) = 0$, then f is continuous at x_0 . If $f(x_0) > 0$, then we use the following inequalities

$$\begin{aligned} \left| |\varrho_n \cos [n(x_0 + h) - \alpha_n]| - |\varrho_n \cos [n(x_0 - h) - \alpha_n]| \right| &\leq 2|\varrho_n \cos (nx_0 - \alpha_n)| \\ \left| |\varrho_n \cos [n(x_0 + h) - \alpha_n]| - |\varrho_n \cos [n(x_0 - h) - \alpha_n]| \right| &\leq 2|\varrho_n| |\sin nh|. \end{aligned}$$

From the first formula it follows that if $f(x_0 + h) = 0$ then $f(x_0 - h) = 0$. If $f(x_0 + h) > 0$ then for every N

$$\begin{aligned} |f(x_0 + h) - f(x_0 - h)| &\leq \sum_{n=1}^{\infty} \left| |\varrho_n \cos [n(x_0 + h) - \alpha_n]| - |\varrho_n \cos [n(x_0 - h) - \alpha_n]| \right| \leq \\ &\leq \sum_{n=1}^N 2|\varrho_n| |\sin nh| + \sum_{n=N+1}^{\infty} |\varrho_n \cos (nx_0 - \alpha_n)|. \end{aligned}$$

It follows easily that f is symmetrically continuous. Obviously f is not continuous at every point where it is positive.

References

- [1] *H. Fried*: Über die symmetrische Stetigkeit von Funktionen, *Fund. Math.* 29 (1937), 134—137.
- [2] *S. Saks*: *Theory of the Integral*, Warszawa 1937.

Author's address: Praha 8 - Karlín, Sokolovská 83 (Matematicko-fyzikální fakulta KU).