Werk

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that $\lambda^2 j - \mathscr{C}^{\cdot}$ is one-to-one, $(\lambda^2 j - \mathscr{C}^{\cdot})^{-1} \in \mathfrak{L}(E)$ and

$$[*] \qquad \left\| \frac{\mathrm{d}^n}{\mathrm{d}\lambda^n} \lambda(\lambda^2 j - \mathscr{C}^{\cdot})^{-1} \right\| \leqq \frac{Mn!}{2} \left(\frac{1}{(\lambda + \omega)^{n+1}} + \frac{1}{(\lambda - \omega)^{n+1}} \right)$$

for every $\lambda > \omega$ and $n = 0, 1, 2, \ldots$

Now a simple computation shows that

$$(\mu j - \mathscr{C}^{\cdot})^{-1} (\alpha_1 e_{2k} + \alpha_2 e_{2k+1}) =$$

$$= \left(\frac{\mu}{\mu^2 + k^2} \alpha_1 + \frac{k}{\mu^2 + k^2} \alpha_2 \right) e_{2k} + \left(\frac{\mu}{\mu^2 + k^2} \alpha_1 - \frac{k}{\mu^2 + k^2} \alpha_2 \right) e_{2k+1}$$

for every $\mu > \omega^2$, $\alpha_1, \alpha_2 \in R$ and $k = 0, 1, 2, \ldots$

This implies $\langle (\mu j - \mathscr{C}^{\cdot})^{-1} (e_{2k}), e_{2k} \rangle = \mu/(\mu^2 + k^2)$ and $\langle \lambda(\lambda^2 j - \mathscr{C}^{\cdot})^{-1} (e_{2k}), e_{2k} \rangle = \lambda^3/(\lambda^4 + k^2)$ for every $\mu > \omega^2$, $\lambda > \omega$ and $k = 0, 1, 2, \ldots$

Since from [*]

$$\left| \left\langle \frac{\mathrm{d}^n}{\mathrm{d}\lambda^n} \lambda(\lambda^2 j - \mathscr{C}^{\cdot})^{-1} (e_{2k}), e_{2k} \right\rangle \right| \leqq \frac{Mn!}{2} \left(\frac{1}{(\lambda + \omega)^{n+1}} + \frac{1}{(\lambda - \omega)^{n+1}} \right)$$

we obtain that

$$\left| \frac{\mathrm{d}^n}{\mathrm{d}\lambda^n} \frac{\lambda^3}{\lambda^4 + k^2} \right| \leqq \frac{Mn!}{2} \left(\frac{1}{(\lambda + \omega)^{n+1}} + \frac{1}{(\lambda - \omega)^{n+1}} \right).$$

But

$$\frac{\lambda^3}{\lambda^4 + k^2} = \int_0^\infty e^{-\lambda\tau} \cos \left[\sqrt{\left(\frac{k}{2}\right)} \tau \right] \coth \left[\sqrt{\left(\frac{k}{2}\right)} \tau \right] \mathrm{d}\tau$$

for every $\lambda > 0$ (cf. the proof of 4,11 in [3]). If we proceed analogously as in the proof of 4,11 in [3] we obtain that $\cos [\sqrt{(k/2)}\, t] \coth [\sqrt{(k/2)}]\, t \leqq M \coth (\omega t)$ for every $t \in R^+$ and $k = 0, 1, 2, \ldots$ But this is evidently impossible for sufficiently large k. We have obtained a contradiction and hence proved 8°.

9° Final part: 7° and 8° establish our theorem.

References

[1] *Béla Szökefalvi-Nagy:* Spektraldarstellung linearer Transformationen des Hilbertschen Raumes, Berlin 1942.

[2] *E. Hille* and *R. S. Phillips:* Functional Analysis and Semi-Groups, Providence 1957.

[3] *M. Sova:* Cosine operator functions, Rozprawy Matematyczne, XLIX, 1966.

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