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в соответствующем полупространстве одна, является точка A_n. Если $A_n \in E_3^+$ (соотв. $\in E_3^-$), то пусть начальной точкой D_n отрезка $F(T_n) \cap R$ является точка пересечения R со стороной $S(A_n, B_n)$ (соотв. $S(A_n, C_n)$); концевую точку отрезка $F(T_n) \cap R$ обозначим через E_n; пусть φ_n – линейное на $\langle 0, 1 \rangle$ отображение, для которого имеет место $\varphi_n(0) = D_n$, $\varphi_n(1) = E_n$.

Тогда P можно разложить на конечное число непересекающихся непустых множеств $P_m (m = 1, \ldots, r,\ r \geqq 0)$ так, что:

Каждое множество P_m можно циклически упорядочить (в статье показано, каким образом это упорядочение можно эффективно осуществить) *так, что если*

$$n(1) \prec n(2) \prec \ldots \prec n(k) \prec n(1) \prec \ldots$$

– все элементы P_m, то

$$\psi_m = \varphi_{n(1)} \dotplus \varphi_{n(2)} \dotplus \ldots \dotplus \varphi_{n(k)}$$

будет замкнутой кривой. Если обозначить через χ систему всех кривых ψ_m и положить

$$\operatorname{ind}_\chi w = \sum_{m=1}^{r} \operatorname{ind}_{\psi_m} w$$

для всех $w \in R - F(\mathbf{B})$, то для этих w справедливо равенство

$$\operatorname{ind}_\chi w = \operatorname{ind}_F w\,.$$

(Индекс точки $w \in R$ относительно кривой в R, а также индекс точки относительно поверхности, был введен в [1].)

Summary

THE INDEX OF A POINT WITH RESPECT TO A POLYHEDRAL SURFACE AND TO ITS PLANE SECTION

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Let $\mathfrak{A} = \{A_i\}_{i=1}^k$ be a (nonempty) finite system of cubes (in E_3) with unit volumes and integer coordinates of vertices. Let $\mathfrak{B} = \{\mathbf{B}_j\}_{j=1}^l$ be the system of all the faces of cubes $\mathbf{A}_i$ which lie on the boundary of $\mathbf{A} = \bigcup_{i=1}^k \mathbf{A}_i$; let $\mathbf{B}$ be their union: $\mathbf{B} = \bigcup_{j=1}^l \mathbf{B}_j$. Let us suppose that the system $\mathfrak{A}$ has the following property: each side of each face from $\mathfrak{B}$ lies in exactly two different faces from $\mathfrak{B}$.

Each ordered triplet $\tau = \{a, b, c\}$ of points from E_3 is called an "*oriented trianqle*"; we say that the triplets $\{a, b, c\}$, $\{b, c, a\}$, and $\{c, a, b\}$ are identical. Let $T(\tau)$ be the convex envelope of the set containing the points a, b, c, let $S(a, b)$ be the convex envelope of the set containing the points a, b. An "*oriented triangle on* $\mathfrak{B}$" is an

arbitrary triplet $\tau = \{a, b, c\}$, the points of which lie on one face $\mathbf{B}_j \in \mathfrak{B}$, and which has the following property: If $\mathbf{B}_j \in \mathfrak{B}$ is the face of the cube $\mathbf{A}_i \in \mathfrak{A}$ and if γ_j is the vector with origin in the center of the cube $\mathbf{A}_i$ and terminal vertex in the center of $\mathbf{B}_j$, then the outer product $[b - a, c - a, \gamma_j]$ is positive.

Each system $\mathfrak{B} = \{\tau_n\}_{n=1}^{p}$ of oriented triangles $\tau_n = \{a_n, b_n, c_n\}$ on $\mathfrak{B}$ having the property that $\mathbf{B} = \bigcup_{n=1}^{p} T(\tau_n)$ and that for $1 \leqq m < n \leqq p$ the intersection of $T(\tau_m)$, $T(\tau_n)$ is either the empty set or the common vertex or the common side of the both triangles, is called "*a trianqulation of the set* $\mathfrak{B}$". A mapping F of the set $\mathbf{B}$ into E_3, linear on each triangle $T_n = T(\tau_n)$, is called "*a polyhedral surface*"; the points a_n, b_n, c_n are the "*vertices of the triangulation* $\mathfrak{B}$", their images $A_n = F(a_n)$, $B_n = F(b_n)$, $C_n = F(c_n)$ are the "*vertices of the surface*".

The main result of the present article is the following theorem: Let R be a plane in which the base $\{X_1, X_2\}$ is given; let us suppose that R does not contain any vertex of the surface F. Let $P = E[n; F(T_n) \cap R \neq \emptyset]$, $Y = X_1 \times X_2$ (vector product). Let E_3^+ (resp. E_3^-) be the open half-space in E_3 whose boundary is R and which contains the point $w + Y$ (resp. $w - Y$), if $w \in R$.

Orient each of the segments $F(T_n) \cap R$ (where $n \in P$) in the following way: One of the half-spaces E_3^+, E_3^- contains only one vertex of $F(T_n)$, the other half-space contans two vertices. Let us suppose (we can do so without restricting generality) that A_n is the vertex of $F(T_n)$ which lies alone in the corresponding half-space.

If $A_n \in E_3^+$ (resp. $A_n \in E_3^-$), let the origin D_n of the segment $F(T_n) \cap R$ be the point of intersection of R with the side $S(A_n, B_n)$ (resp. $S(A_n, C_n)$); let E_n be the terminal point of the segment $F(T_n) \cap R$. Let φ_n be the linear mapping of the interval $\langle 0, 1 \rangle$, for which $\varphi_n(0) = D_n$, $\varphi_n(1) = E_n$.

Then we can decompose the set P into a finite number of disjoint nonempty sets P_m $(m = 1, \ldots, r;\ r \geqq 0)$ so that the following holds:

Each set P_m can be ordered cyclically (in the article there is shown how this can be performed effectively) in such a manner that if

$$n(1) \prec n(2) \prec \ldots \prec n(k) \prec n(1) \prec \ldots$$

are all the elements of P_m, then

$$\psi_m = \varphi_{n(1)} \dotplus \cdots \dotplus \varphi_{n(k)}$$

is a closed curve. If we denote the system of all the curves ψ_m by the symbol χ and if we put

$$\operatorname{ind}_\chi w = \sum_{m=1}^{r} \operatorname{ind}_{\psi_m} w \text{ for all points } w \in R - F(\mathbf{B}),$$

then the following relation holds:

$$\operatorname{ind}_\chi w = \operatorname{ind}_F w \text{ for all } w \in R - F(B).$$

(The index of a point $w \in R$ with respect to a curve in R and the index of a point with respect to a surface are defined in [1].)