Summary

A NOTE ON PERIMETER OF THE CARTESIAN PRODUCT OF TWO SETS

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Denote by $\mathscr{V}$ the system of all infinitely differentiable vector-valued functions $v = [v_1, \ldots, v_k]$ on E_k with $v(z) = 0$ for sufficiently large $|z|$. Further, let $\mathscr{V}^1$ be the system of all $v \in \mathscr{V}$ with $\max |v(z)| \leqq 1$, $z \in E_k$. For every Lebesgue measurable set $C \subset E_k$ the perimeter $\|C\|$ of C is defined by

$$\|C\| = \sup_{v \in \mathscr{V}^1} \int_C \operatorname{div} v(z) \, dz .$$

It is known that $\|C\| < \infty$ is a necessary and sufficient condition for the existence of a finite Borel measure P_C over the boundary H_C of C and a Borel measurable vector-valued function $v^C = [v_1^C, \ldots, v_k^C]$ on H_C such that

$$\text{(i)} \qquad v \in \mathscr{V} \Rightarrow \int_{H_C} v \,.\, v^C \, dP_C = \int_C \operatorname{div} v(z) \, dz ,$$

$$\text{(ii)} \qquad |v^C(z)| = 1 \quad \text{for } P_C - \text{almost every } z \in H_C$$

((i), (ii) determine P_C uniquely and v^C almost uniquely with respect to P_C). Suppose now that $k = r + s$, where r, s are positive integers. Denote by L_m the m-dimensional Lebesgue measure. It is proved that, for arbitrary Lebesgue measurable sets $A \subset E_r$, $B \subset E_s$, the formula

$$\|A \times B\| = \|A\| \,.\, L_s B + L_r A \,.\, \|B\|$$

is true. In particular, $\|A \times B\| < \infty$ whenever $\|A\| + \|B\| + L_r A + L_s B < \infty$; in the latter case the structure of the measure P_C, corresponding to $C = A \times B$, is simply described by means of $P_A \times L_s$, $L_r \times P_B$.