# Measurement of Permeability and Ferromagnetic Resonance at Microwave Frequencies.

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Two new methods are described for measurement of permeability of ferromagnetic substances in the centimeter-wave range. The first one is suitable for thin films and for a moderately wide frequency band, the second one is used for thin wires and for the lower end of the centimeter-wave band (below 10 cm). The last part of the paper contains some permeability data obtained for nickel films at wavelengths 12,4 and 3,20 cm and for nickel wires a wavelength of 3,20 cm in super imposed d. c. magnetic field (ferromagnetic resonance).

I. *Introduction.* The dispersion of permeability in the centimeter-wave band is of great value for understanding the elementary magnetisation processes and has been therefore extensively studied for a long time. An exhaustive survey of all experimental methods and results has been given by Allanson.[1]) The theoretical aspects of the problem have been recently discussed by Kittel.[2]) In despite of the great number of papers published until now the results of individual investigations are not always in a satisfactory agreement and the question of the nature of the dispersion does not seem to be settled. For this reason there is still a need for further measurements especially as concerns the newly discovered effect of so called ferromagnetic resonance in d. c. magnetic fields. We have devised and tried out two different methods for measurements on thin films and wires, both based on the existence of skin-effect and giving thus the resistive part $\mu_R$ of permeability.

II. *Measurements on thin films.* The method is based upon the shielding effect of a film placed between two inductively coupled loops (Fig. 1). The film is produced by evaporating the metal in a high vacuum on a supporting glass plate. The permeability is determined from the shielding effect of the investigated ferromagnetic film compared with that of a nonferromagnetic one.

[1]) Allanson: The permeability of ferromagnetic materials at high frequencies between $10^5$ and $10^{10}$ c/s. J. Inst. El. Eng. **97** (III), 247 (1945).

[2]) C. Kittel: Theory of dispersion of permeability etc. Phys. Rev. **79**, 281 (1946).

The shielding effect depends upon $\mu$, $\varrho$, $d$ and $\omega$ ($\mu$ being permeability, $\varrho$ specific resistance, $d$ thickness and $\omega$ cyclic frequency) and upon the geometrical configuration of the measuring system. To eliminate this latter factor all measurements must be made under the same geometrical configuration. The distance between the loops and the film must be kept constant, all specimens must have the

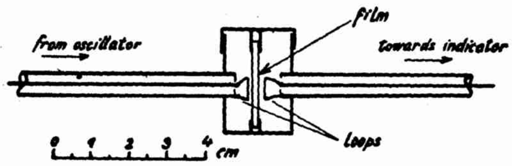


Fig. 1. Arrangement for measurement of the shielding effect of thin films.

same lateral dimensions, the loops must be small as compared with the wavelength and there must be no stray coupling between the loops. All this must be guaranteed by an appropriate mechanical construction. Under these suppositions we can find the conditions under which two films of different materials possess the same shielding effect.

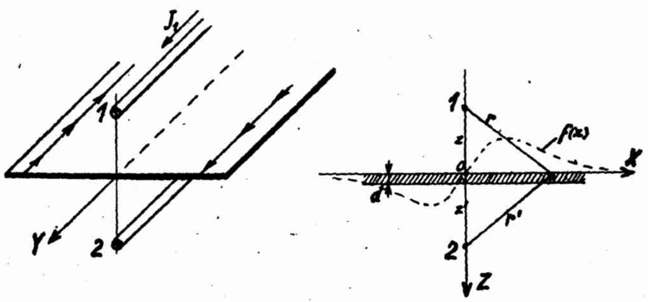


Fig. 2. Currents induced in the film and definition of the coordinate system.

Let us suppose that both the straight parts of the loops are extremely thin as compared with their length and the distance from the film. Further that the surface of the film is very large and its thickness very small as compared with the distances $z$ and $z'$, and the coordinates defined according to the Fig. 2.

The shielding effect of the film may then be interpreted as follows: The current $I_1$ flowing through the wire *1* is inducing another current in the wire *2* and on the other hand induces linear currents within the film, parallel to the axis $Y$. The latter currents produce a magnetic field which is opposite to the effect of the primary field. Let us suppose further that the thickness and permeability of the film are so small that its static magnetic moment may be neglected. When we denote the $x$-component of the field produced by the currents in the film by $H_{2x}$ and the $x$-component of the field in the place of the wire *2* produced by the current $I_1$ in absence of the film by $H_{1x}$ we can define the transmission factor of the film as fraction

$$\eta = H_{1x}/H_{2x}.$$

The magnetic field have only the $x$- and $z$-components $H_x$, $H_z$ the electric field only the component $E_y$.

The primary magnetic field $H_1$ inside the film is a function of $x$, $z$ and $t$ (time) of a form

$$\begin{aligned} H_1 &= \exp\,[\jmath\omega t - \gamma z]\,.\,f(x), \\ \gamma &= (1 + \jmath)\,c^{-1}\sqrt{2\pi\mu\omega/\varrho}. \end{aligned} \tag{1}$$

The function $f(x)$ decribes in the plane $z = 0$ the decrease of the field with the distance $x$ and is given only by the geometry of the problem. The intensity of the electric field inside the film at a distance $x$ follows from Maxwells equation

$$\int E\,\mathrm{d}s = -\frac{\mu}{c}\frac{\mathrm{d}}{\mathrm{d}t}\int H\,\mathrm{d}p\,.$$

(omitting the periodic time factor)

$$E_y = -\,c^{-1}\jmath\omega\mu\,.\,\exp\,[-\,\gamma z]\int_0^x f(x)\,\mathrm{d}x$$

and the current density inside the film is

$$i = -\,c^{-1}\jmath\omega\mu\sigma\,.\,\exp\,[-\,\gamma z]\int_0^x f(x)\,\mathrm{d}x,$$

$\sigma$ being the specific conductivity of the film. The total effect of all elementary linear currents in the film builds up — according to the Biot-Savart's law — the secondary field $H_2$ in the place of the wire *2*. Its $x$-component is:

$$H_{2x} = \int_{-\infty}^{\infty}\int_0^d \frac{2i}{r'}\cdot\frac{z'}{r'}\,\mathrm{d}x\,\mathrm{d}z =$$

$$= -\,c^{-1}\,.\,2\jmath\omega\mu\sigma\int_{-\infty}^{\infty}\int_0^d \exp\,[-\,\gamma d]\,\frac{z'\,f(x)}{(x^2 + z'^2)}\,\mathrm{d}x\,\mathrm{d}z =$$

$$= \frac{2\jmath\omega\mu\sigma}{c\gamma}\,(1 - \exp\,[-\,\gamma d])\int_{-\infty}^{\infty}\frac{z'\,f(x)}{(x^{\cdot\cdot} + z'^2)}\,\mathrm{d}x\,\mathrm{d}z.$$

By using eq. (1) and supposing $\gamma d \ll 1$ we obtain finally

$$H_{2x} = \frac{c}{2\pi} \gamma^2 d \int_{-\infty}^{\infty} F(x)\, \mathrm{d}x. \tag{2}$$

The value of the integral on the right side depends only upon the kind of the function $F(x)$. In every case it is a constant depending upon the geometry of the problem (here upon $z$ and $z'$). Under the assumption of negligible static magnetic moment of the film the field $H_{1x}$ is also only a function of the distance $z + z'$ and thus $\eta$ depends only upon $\gamma^2 d$.

When there are two films which under the same geometrical configuration and the same frequency possess the same transmission factor $\eta$ then it must be

$$\gamma_1^2 d_1 = \gamma_2^2 d_2$$

or

$$\sigma_1 \mu_1 d_1 = \sigma_2 \mu_2 d_2.$$

Comparing a ferromagnetic film ($\mu_1 = \mu$) with a nonferromagnetic one ($\mu_2 = 1$) we have:

$$\mu = \sigma_2 d_2 / \sigma_1 d_1 = \varrho_1 d_2 / \varrho_2 d_1. \tag{3}$$

III. *Measurements on thin wires*. In the case of a wire of circular cross section the relative increase of resistance $\vartheta = R/R_0$ ($R_0$ being the d. c. resistance) is a function of the argument

$$\xi = \sqrt{\pi \mu \sigma f}, \qquad \vartheta = \vartheta(\xi). \tag{4}$$

The transcendental function $\vartheta$ is well known and tabulated for example in the Bureau of Standards Circular No. 74.

The measurement of $\vartheta$ is performed by means of an arrangement sketched on Fig. 3. Across a waveguide is stretched a ferromagnetic wire together with a nonmagnetic wire used for comparison. Both are parallel to the direction of the electric field and the high frequency voltage applied across them is the same. Their distance is so great that their mutual influence may be neglected. The wires are used as bolometers. For separating the d. c. circuits the waveguide section is longitudinally cut along the middle plane and the plate with the common point of the wires is capacitively separated from the waveguide.

Let us denote $R$ and $R_0$ the high frequency and d. c. resistances of the ferromagnetic wire, $R'$ and $R'_0$ the same for the nonmagnetic wire. If $I$ and $I'$ are the high frequency currents flowing in the wires and $I_0$, $I'_0$ the d. c. calibrating currents producing the same heating of the wires we may write

$$\begin{aligned} RI^2 &= R_0 I_0^2 \\ R'I'^2 &= R'_0 I'^2_0 \\ RI &= R'I'. \end{aligned} \tag{5}$$

Then we obtain for $\vartheta$ of the ferromagnetic wire

$$\vartheta = \frac{R}{R_0} = \left(\frac{I'_0}{I_0}\right)^2 \frac{R'_0 R'}{R_0^2}. \tag{6}$$

The currents $I_0$ and $I'_0$ are obtained from the d. c. calibrating curves of the wires, $R'_0$ and $R_0$ are known and $R'$ (for the nonmagnetic wire is $\mu = 1$) is calculated by means of eqs. (4) and (5). To the $\vartheta$ obtained in this way we find corresponding $\xi$ in the tables and from this

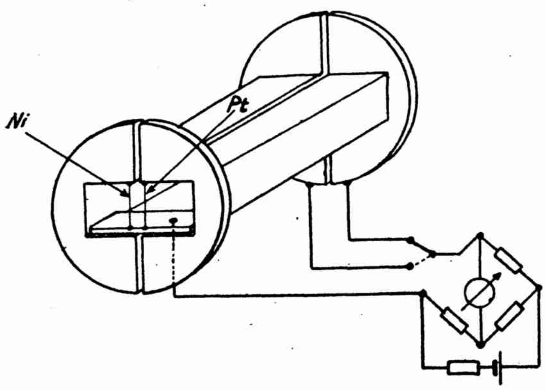


Fig. 3. Arrangement of a ferromagnetic and a nonmagnetic bolometer wire in a waveguide section.

$$\mu = \frac{\xi^2 \, . \, R_0}{0{,}255 \, . \, f \, . \, l}, \tag{7}$$

$f$ being frequency im $Mc/s$, $l$ length of the wire in cm.

In the formula (6) the difference between uniform distribution of the d. c. along the wire and a sinusoidal distribution in the case of high frequency current is neglected. Also the cooling effect at the ends of the wire is neglected. Appropriate corrections would appear as factors at $I_0$ and $I'_0$ which would have the same value (and therefore cancel each other) in the case that both wires have not very different resistance, diameter, length and thermal conductivity.

IV. *Experimental results for nickel.* First measurements have been made on nickel as there have been at our disposal very pure specimens of vacuum annealed nickel wires and strips from Heraeus (99,8% Ni, less than 0,2% Mn).

Measurements on films have been made with an apparatus according to Fig. 1 and on a low power level (magnetron oscillators for 3,2 and 12,4 cms of about 10 to 50 mw output, respectively). Two sets of films have been prepared by evaporation of nickel and silver in vacuo on glass plates 30 mm in dia. and 1,5 mm thick. The thickness of the films ranging from about 500 to 3000 Å has been determined by means of an interferometer. For a given wavelength there have been selected from both sets two films showing the same transmission factor $\eta$, and then $\mu$ has been computed from the eq. (3). As the conductivity $\sigma$ for evaporated films differs from that of the massive metal and depends upon the thickness it had to be determined for each specimen separately.

It appeared however, that the static permeability of the films depends upon the thickness and decreases towards unity for very thin films (Fig. 4). The results obtained with this method cannot therefore be considered fully valid for solid nickel. In order to approach closer to the case of solid metal the thickness should be at least 10 000 Å. The transmission factor $\eta$ would be then very low and its measurement would become very inaccurate.

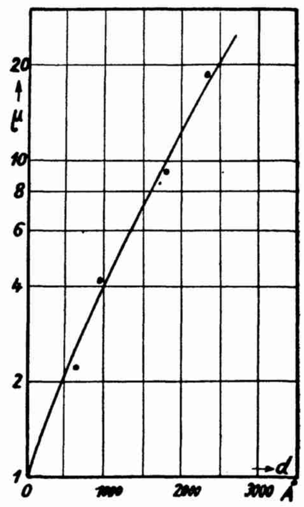

Fig. 4. Static permeability of nickel films as a function of thickness.

The results are as follows:

For 12,4 cms wavelength: $d_{Ag} = 1050$ Å, $\varrho_{Ag} = 0{,}28 \,.\, 10^{-4}\, \Omega \,.\, \mathrm{cm}$, $d_{Ni} = 2950$ Å, $\varrho_{Ni} = 1{,}80 \,.\, 10^{-4}\, \Omega \,.\, \mathrm{cm}$, $\eta_{Ag} = \eta_{Ni} = 0{,}21$, $\mu = 2{,}3$. For 3,20 cms wavelength: $d_{Ag} = 460$ Å, $\varrho_{Ag} = 2{,}5 \,.\, 10^{-4} \Omega \,.\, \mathrm{cm}$, $d_{Ni} = 640$ Å, $\varrho_{Ni} = 5{,}1 \,.\, 10^{-4}\, \Omega \,.\, \mathrm{cm}$, $\eta_{Ag} = \eta_{Ni} = 0{,}12$, $\mu \doteq 1{,}4$.

Ferromagnetic resonance effect in superimposed d. c. magnetic field has not been found in any nickel film between 500 to 2000 Å in thickness.

Measurements on nickel wires by the second method have been performed with an arrangement according to the Fig. 3. The waveguide section ($13 \times 28$ mm) has been coupled by means of a matching device to the output of a 725 — A — type pulsed magnetron working in 3-cm wave — band. Changes in the d. c. resistance of the wires have been measured by means of a General-Radio Co Bridge type 650 — A, with external galvanometer. Nickel wires had diameters from 0,03 to 0,07 mm, platinum wires about 0,01 to 0,02 mm.

The results for 3,20 cm wavelength and for a nickel wire 0,04 mm dia. are: $I_0 = 0{,}096$ amp., $R_0 = 1{,}91$ ohm, $l = 11{,}2$ mm. Platinum wire: 0,01 mm dia., $I'_0 = 0{,}035$ amp., $R'_0 = 11{,}7$ ohm, $R' = 23{,}4$ ohm (computed), $l = 11{,}1$ mm. From there numbers follows: $\vartheta = R/R_0 = 9{,}40$, $\xi = 25{,}8$ and $\mu = 4{,}60$.

The bridge current through the nickel wire was less than 5 ma. High frequency current did not cause a magnetic field strength on the surface of the wire greater than 10 gauss.

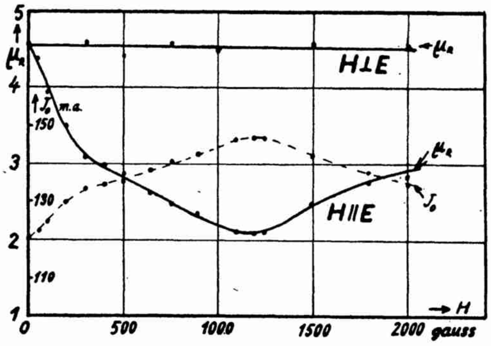


Fig. 5. Ferromagnetic resonance effect in nickel wire at a wavelength of 3,20 cm with magnetic field paralel and perpendicular to the axis of the wire.

With the same arrangement an attempt has been made to measure the ferromagnetic resonance effect discovered by Griffiths.[3] [4]) The results are shown in Fig. 5. The end with the wires has been placed between the poles of an electromagnet with the wires either parallel or at a right angle to the magnetic field. The effect has been found only in the first case. In the second case there is no influence of the d. c. field on the permeability.

As the high-frequency current in the nickel wire increased with the field although the potential difference voltage across the ends was constant it must be concluded that the high frequency resistance was diminished. Therefore the permeability decreases with the field, contrary to the results obtained by the $Q$-method by Griffiths. The resonance field strength (about 1100 gauss) is, however, in a good agreement with the value found by Griffiths.

[3]) Griffiths: Anomalous high-frequency resistance of ferromagnetic metals; Nature, 158, 670, 1946.

[4]) Kittel: Interpretation of anomalous Larmor frequencies in ferromagnetic resonance experiments; Phys. Rev. 71, 270, 1946.