A precise method of determining the constant of crystal grating by the combination of φ- and $\varkappa$-methods.

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A new type of spectrograph has been constructed. Its principal parts are three coaxial cylinders of chrome-nickel-steel, which are made by Skoda-Works. The grating constant of a zinc sulphide crystal has been measured with this spectrograph using the method of Siegbahn to verify the advantages of the $\varkappa$-methods against the φ-method as shown by the former authors. It has been shown that the defects of the crystal such as displacement can be determined and eliminated by the combination of the φ- and the $\varkappa$-methods. The possibility of the said determination is given by the fact that the value of the constant of crystal grating derived through the $\varkappa$-method can be, in the first approximation, taken to be the correct value. After determining the said error of displacement, even the minute error of the measured angle $\varkappa$ can be eliminated. The said method has been experimentally verified.

In this journal two precise methods of determining the grating constant of a crystal have been published. These methods do not require to measure the glancing angle φ, as demanded by the Bragg's law, but they need to measure and utilise the angle $\varkappa$, the diffrence between two glancing angles. We should like to point out that the angle $\varkappa$, in either one of these methods, is only measured with the Siegbahn's precise method for the measurement of the glancing angle φ.

Of these two $\varkappa$-methods, one, evolved by Kunzl and Köppel,[1]) measures in the manner of Pavelka,[2]) the difference between the glancing angles of the same radiation in two different orders m and n, i. e. $\varkappa = \varphi_n - \varphi_m$, while the other, evolved by Bouchal and Dolejšek,[3]) measures in a manner similar to Valouch,[4]) the difference between the glancing angles of two different radiations λ_μ

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[1]) V. Kunzl-J. Köppel, C. R. **196** (1933), 787; **196** (1933), 940; Časopis **63** (1934), 109; Journ. de Phys. **5** (1934), 145.

[2]) A. Pavelka, Bull. de l'Acad. de Sc. de Bohême **28** (1927), 442.

[3]) F. Bouchal-V. Dolejšek, C. R. **199** (1934), 1054; Časopis **65** (1935), 34.

[4]) M. A. Valouch, Bull. de l'Acad. de Sc. de Bohême **28** (1927), 31.

and λ_ν in one and the same order, i. e., $\varkappa = \varphi_{n,\nu} - \varphi_{n,\mu}$. Such is the difference between these methods so far as their fundamentals are concerned. Again if we look at the specific advantages, which have been shown in the cited papers, the difference between them is even greater. In the Kunzl-Köppel's method the fictive constants vary in accordance with the radiations used and they naturally differ from those of Bragg. In the series of the fictive constants derived by this method, which vary according to the radiations, a particular fictive constant, corresponding to a particular wave-lenght, can be found which agrees with the real grating constant. In the method of Bouchal-Dolejšek, the fictive constants do not vary with the wave-lengths used and they agree with the fictive constants derived from the φ-method of Bragg. In this method any two wave-lengths can be so chosen as to make the angle $\varkappa$ much smaller than in the Kunzl-Köppel's method, and through such a selection of the small angle $\varkappa$, it is possible to enlarge the advantage of the other one where the value of the angle $\varkappa$ is nearly as great as that of the angle φ. We cannot however make a right use of this additional advantage for in the region of small angles the discrepancy of the scale has great influence on the results.

In this work, in verifying the mentioned advantages of the $\varkappa$-methods of determining the grating constant of a crystal given by the former authors, we develop a new method by the combination of the φ- and $\varkappa$-methods for entirely eliminating the discrepancy due to the defects of the grating crystal (such as displacement etc.), there by avoiding the necessity of selecting the angle $\varkappa$ which is small. Here, in applying this method, we measure and give out the precise value of the grating constant of a zinc sulphide (ZnS) crystal as its verification.

We have chosen zinc sulphide crystal as a diffraction grating, since it has, owing to small grating constant,[5]) large dispersion, and at the same time has, as shown by Küpferle,[6]) a good reflecting power. Therefore it is of great value as a diffraction grating for X-rays. From the other side, the natural surface of this crystal is very usually not quite regular and consequently it is not possible to adjust the forefront surface of the crystal exactly at the centre of the spectrograph with the usual optical methods. It can be noted that even with such an irregular surface of the crystal, we can with the $\varkappa$-method derive precise value of the grating constant and thereby demonstrate the special merit and advantage of the $\varkappa$-method.

For our proposed work we have used a new spectrograph (Fig. 1) which we have constructed. In its principle parts, this spectrograph consists of three coaxial cylinders. These cylinders

[5]) A. Pavelka, l. c.

[6]) G. Küpferle, ZS. f. Phys. **98** (1935), 237.

are made of chrome-nickel-steel by Skoda-Works in Plzeň (Czechoslovakia). They are cut and ground so as to fit in one another so precisely that they do not admit cavity or looseness amidst them more than 1 to 2μ. This is of course the highest precision that is possible to obtain. The precision of our measurements, is however

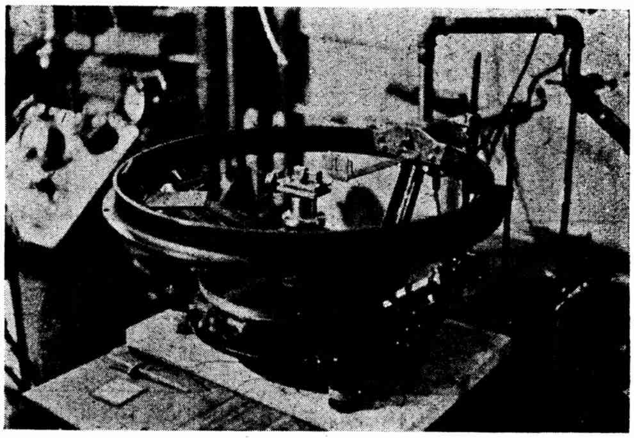

Fig. 1.

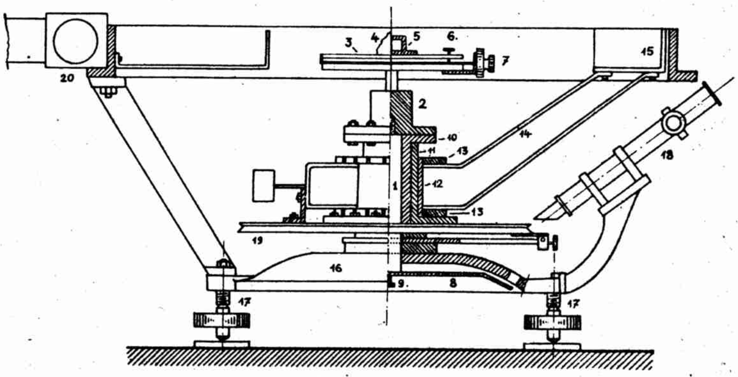

Fig. 2.

limited by conditions other than those of the cylinders as it can be noted from our readings and results. Through the choice of cylinders instead of conical axles the possibility of the eccentricity of the inner and outer cylinders is diminished. The material of the cylinders is such that it does not permit rubbing in due to friction so as to stop the motion of the cylinders. The middle cylinder is

Ist. Series with a displacement of 0,401 mm.

Table 1a. Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta t°$ C	φ 18° C	mean value of φ 18° C
227	0,322	4′ 24,3″	94° 44′ 47,5″	23° 40′ 5,8″	7,07″	23° 40′ 12,87″	23° 40′ 11,9″
228	0,757	10′ 21,3″	94° 50′ 47″	23° 40′ 6,4″	6,78″	23° 40′ 13,18″	
230	0,768	10′ 30,4″	94° 50′ 52″	23° 40′ 5,4″	7,3″	23° 40′ 12,7″	
231	0,329	4′ 30,0″	94° 44′ 48″	23° 40′ 4,5″	7,07″	23° 40′ 11,6″	
233	0,256	3′ 30,1″	94° 36′ 51,5″	23° 40′ 5,4″	5,85″	23° 40′ 11,25″	
234	0,260	3′ 33,4″	94° 36′ 53,5″	23° 40′ 6,7″	6,31″	23° 40′ 13,01″	
235	0,264	3′ 36,7″	94° 36′ 50″	23° 40′ 6,6″	6,38″	23° 40′ 12,98″	
236	0,266	3′ 38,3″	94° 36′ 47,75″	23° 40′ 6,5″	6,61″	23° 40′ 13,1″	
237	0,351	4′ 48,1″	94° 44′ 50″	23° 40′ 0,5″	6,14″	23° 40′ 6,6″	

Table 1b. Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta t°$ C	φ 18° C	mean value of φ 18° C
207	0,2000	2′ 44,2″	42° 27′ 10,5″	10° 36′ 6,6″	0,86″	10° 36′ 7,46″	10° 36′ 8,6″
208	0,1890	2′ 35″	42° 27′ 12,5″	10° 36′ 9,5″	0,91″	10° 36′ 10,41″	
210	0,3980	5′ 26,9″	42° 29′ 52″	10° 36′ 6,25″	1,24″	10° 36′ 7,49″	
211	0,4076	5′ 34,5″	42° 29′ 52″	10° 36′ 4,25″	1,12″	10° 36′ 5,37″	
212	0,4018	5′ 29,8″	42° 29′ 53,5″	10° 36′ 6,0″	0,86″	10° 36′ 6,86″	
213	0,1708	2′ 20,2″	42° 22′ 2,0″	10° 36′ 5,5″	1,31″	10° 36′ 6,81″	
214	0,1762	2′ 24,6″	42° 22′ 7″	10° 36′ 7,9″	1,67″	10° 36′ 9,57″	
238	0,197	2′ 41,5″	42° 22′ 8″	10° 36′ 12,3″	1,11″	10° 36′ 13,41″	
239	0,166	2′ 16,3″	42° 22′ 9″	10° 36′ 6,3″	1,13″	10° 36′ 7,43″	
240	0,187	2′ 33,5″	42° 22′ 7″	10° 36′ 10,1″	1,24″	10° 36′ 11,34″	

Table 1c. Cu — Mo, $\varkappa$

Plate No	Δ mm	Δ'	α	$\varkappa$	$\Delta t°$ C	$\varkappa$ 18° C	mean value of $\varkappa$ 18° C
215	0,3450	4′ 43,3″	26° 12′ 43,5″	13° 4′ 0,1″	1,66″	13° 4′ 1,76″	13° 4′ 2,37″
216	0,3381	4′ 47,5″	26° 12′ 47,4″	13° 3′ 59,95″	1,37″	13° 4′ 1,32″	
217	0,9536	13′ 3,3″	25° 54′ 53,0″	13° 3′ 58,15″	1,79″	13° 3′ 59,94″	
218	0,9690	13′ 15,4″	25° 54′ 53″	13° 4′ 4,2″	1,78″	13° 4′ 5,98″	
219	0,218	2′ 58,9″	26° 5′ 0,5″	13° 3′ 59,7″	1,46″	13° 4′ 1,16″	
220	0,2160	2′ 57,3″	26° 5′ 0,0″	13° 3′ 58,65″	1,69″	13° 4′ 0,34″	
221	0,2240	3′ 3,9″	26° 11′ 7,0″	13° 4′ 1,55″	1,878″	13° 4′ 3,43″	
223	0,2190	2′ 59,8″	26° 11′ 4,5″	13° 4′ 2,35″	2,204″	13° 4′ 4,55″	
224	0,2072	2′ 49,1″	26° 5′ 5″	13° 3′ 57,05″	2,2″	13° 3′ 59,25″	
225	0,216	2′ 57,3″	26° 5′ 8″	13° 4′ 2,65″	2,088″	13° 4′ 4,74″	
226	0,228	3′ 7,1″	26° 11′ 8,5″	13° 4′ 0,7″	2,92″	13° 4′ 3,62″	

Table 1d.

Displacement of the crystal......	$= 0{,}401$ mm
φ_1Cu........................	$= 23^\circ\, 40'\, 11{,}9''$
φ_1Mo........................	$= 10^\circ\, 36'\, 8{,}6''$
$\varphi_1\text{Cu} - \varphi_1\text{Mo} = \varkappa$	$= 13^\circ\, 4'\, 3{,}3''$
Directly measured $\varkappa$	$= 13^\circ\, 4'\, 2{,}37''$
Mean $\varkappa$	$= 13^\circ\, 4'\, 2{,}84''$

fixed to the body of the spectrograph. The inner cylinder forms the axle of the crystal table and the outer cylinder that of the cassette holder. The construction of the spectrograph, which can be seen from the sketch (Fig. 2), is besides having the said cylinders instead of cones also different only in certain details from the usual spectrograph of the Siegbahn's type. This is of course due to the fact that the steel of the cylinders after hardening once, becomes so hard that it is no more possible to work on it. The cylinders have been ground at a temperature of 20° C. No influence of higher temperature on the precision of our readings has been found in the summer days. But if the temperature is less than 20° C, it is rather difficult to rotate either the crystal or the cassette. Before we discuss our readings with the mentioned spectrograph, we may say that the accuracy of the readings is limited by the scale.

It is with the above stated spectrograph that we have taken a set of three series of measurements, one of the glancing angle φ of Cu $K\alpha_1$, one of those of the glancing angle φ of Mo $K\alpha_1$ and another series of those of the angle $\varkappa$ (Cu $K\alpha_1$ — Mo $K\alpha_1$), which can be seen from the Table 1 *a*, *b*, *c*, and *d*. First if we consider the series of measurements of the glancing angle φ of Cu $K\alpha_1$, excepting the last of the series, there is only a difference of 1, 5 or 2 seconds between the highest and the lowest value. In the case of the series of measurements of the φ of Mo $K\alpha_1$, there is slightly a greater difference between the maximum and the minimum. This precision is satisfactory for our work. In this connection, we wish to point out that the angle $\varkappa$, derived from the mean value of the glancing angle φ of Cu $K\alpha_1$ and that of the glancing angle φ of Mo $K\alpha_1$, agrees with the mean value of the measured angle $\varkappa$ with only a difference of nearly 0,9″. This agreement can be taken as a test of the accuracy of our measurements.

We have taken, as we have already mentioned, zinc sulphide crystal with a natural cut surface, which has given clear and well defined spectral lines. The crystal has however been slightly curved. We therefore could not fulfil the necessary conditions of adjustment of the crystal with the usual optical methods. Hence there has been actually a certain displacement which has introduced a discrepancy in the value of the glancing angle φ of Cu $K\alpha_1$ and that of the glancing angle φ of Mo $K\alpha_1$ and consequently in the value of the grating constant of the crystal. How the displacement of the crystal affects the values of the glancing angles and consequently the value of the grating constant of the crystal derived from them, can be seen from our Table 2. When we see the first series of measurements which are taken with one and the same condition of the crystal-adjustment we find that the values of the constant of crystal grating derived from φ of Cu $K\alpha_1$ and φ of Mo $K\alpha_1$ are at

Table 2.

Grating constant of ZnS crystal at a temperature of 18° C.

I ... Ist series of measurements with a displacement $\Delta = 0{,}401$ mm
II ... II nd series of measurements with a displacement $\Delta = 0{,}025$ mm

d_1 XU	I	II	difference	mean corr. value
φ_1CU	1914,77	1908,6		
mean $\varkappa$	1908,166	1908,985		
corr. $\varkappa$	*1908,978*	*1908,935*	0,043	1908,96
φ_1Mo	1923,53	1908,0		

great variance with one another. Applying the value of the angle $\varkappa$ and the values of the wave lengths of Mo $K\alpha_1$ and Cu $K\alpha_1$ in the Bouchal-Dolejšek's equation,[7])

$$d_n = \tfrac{1}{4}n\left[\left(\frac{\lambda_\nu - \lambda_\mu}{\sin\frac{1}{2}\varkappa}\right)^2 + \left(\frac{\lambda_\nu + \lambda_\mu}{\cos\frac{1}{2}\varkappa}\right)^2\right]^{\frac{1}{2}}$$

(where d is the constant of crystal grating, n is the order at which the radiation is reflected, λ_ν is the wave length of the radiation ν, λ_μ is the wave length of the radiation μ which is smaller than λ_ν) we have derived 1908, 166 X. U. as the value of the grating constant of the crystal. This value thus derived from the angle $\varkappa$ though differs very much from both the values derived from the φ of Cu $K\alpha_1$ and the φ of Mo $K\alpha_1$ is, as we show at a latter stage, very near to the correct value. But in the second set of three series of measurements, shown in the Table 3, a, b, c and d, which are taken with the crystal-adjustment improved and made better than before by a process which we mention later on, the variance in the values of the constant of crystal grating derived from each one of the three said data (φ of Cu $K\alpha_1$, φ of Mo $K\alpha_1$ and $\varkappa$ of Cu — Mo) is greatly diminished. In the case of the first set of measurements with bad adjustment of the crystal the difference between the maximum and the minimum of the three said values of the constant of crystal grating is nearly 15,5 X. U., while in the case of the

[7]) F. Bouchal and V. Dolejšek, C. R. **199** (1934), 1054.

II nd. Series with a displacement of 0,0245 mm.

Table 3a. Cu, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta t°$ C	φ 18° C	mean value of φ 18° C
256	0,338	4′ 37,4″	94° 54′ 56″	23° 44′ 53,35″	4,31″	23° 44′ 57,66″	23° 45′ 4,346°
260	0,386	5′ 16,8″	94° 54′ 56″	23° 45′ 3,2″	2,38″	23° 45′ 5,58″	
261	0,182	2′ 29,4″	95° 2′ 17,5″	23° 44′ 57,025″	2,27″	23° 44′ 59,52″	
262	0,134	1′ 50″	95° 2′ 19″	23° 45′ 7,125″	2,97″	23° 45′ 10,095″	
265	0,423	5′ 47,2″	94° 55′ 1,5″	23° 45′ 12,05″	1,28″	23° 45′ 13,33″	
266	0,402	5′ 30″	94° 54′ 49″	23° 45′ 4,75″	2,15″	23° 45′ 6,9″	
267	0,203	2′ 46,6″	95° 2′ 50″	23° 45′ 0,85″	1,92″	23° 45′ 2,77″	
268	0,209	2′ 51,5″	95° 2′ 50″	23° 44′ 57,025″	1,89″	23° 44′ 58,91″	

Table 3b. Mo, φ_1

Plate No	Δ mm	Δ'	α	φ_t	$\Delta t°$ C	φ 18° C	mean value of φ 18° C
258	0,247	3′ 22,7″	42° 41′ 51″	10° 41′ 18,425″	0,999″	10° 41′ 19,424″	10° 41′ 22,94″
259	0,265	3′ 37,5″	42° 41′ 51″	10° 41′ 22,125″	0,824″	10° 41′ 22,949″	
263	0,192	2′ 37,6″	42° 48′ 6,5″	10° 41′ 22,225″	1,399″	10° 41′ 23,624″	
264	0,182	2′ 29,4″	42° 48′ 9,5″	10° 41′ 25,25″	0,799″	10° 41′ 26,049″	
269	0,263	3′ 35,9″	42° 41′ 52,5″	10° 41′ 22,1″	0,949″	10° 41′ 23,049″	
270	0,247	3′ 22,7″	42° 41′ 52,5″	10° 41′ 18,8″	0,824″	10° 41′ 19,624″	
271	0,255	3′ 29,3″	42° 41′ 55,5″	10° 41′ 21,2″	0,999″	10° 41′ 22,199″	
272	0,172	2′ 21,2″	42° 47′ 56″	10° 41′ 23,7″	1,149″	10° 41′ 24,849″	
273	0,175	2′ 23,6″	42° 47′ 57,0″	10° 41′ 23,35″	1,249″	10° 41′ 24,699″	

Table 3c. Cu — Mo, $\varkappa$

Plate No	Δ mm	Δ'	α	$\varkappa$	$\Delta t°$ C	$\varkappa$ 18° C	mean value of $\varkappa$ 18° C
274	0,269	3′ 40,8″	26° 10′ 36,5″	13° 3′ 27,85″	1,696″	13° 3′ 29,546″	13° 3′ 39,778″
275	0,247	3′ 22,7″	26° 10′ 31″	13° 3′ 34,15″	1,862″	13° 3′ 36,012″	
276	0,313	4′ 16,9″	26° 3′ 3,5″	13° 3′ 40,2″	2,236″	13° 3′ 42,436″	
277	0,306	4′ 11,2″	26° 3′ 6,5″	13° 3′ 40,928″	2,078″	13° 3′ 40,928″	
278	0,194	2′ 39,2″	26° 9′ 59,0″	13° 3′ 39,9″	1,862″	13° 3′ 41,762″	
279	0,182	2′ 29,0″	26° 9′ 54,5″	13° 3′ 42,75″	1,330″	13° 3′ 44,080″	
280	0,302	4′ 7,9″	26° 3′ 10,5″	13° 3′ 39,2″	1,996″	13° 3′ 41,196″	
281	0,307	4′ 12,0″	26° 3′ 11″	13° 3′ 41,5″	0,768″	13° 3′ 42,268″	

Table 3d. Displacement of the crystal...... = 0,0245 mm
φ_1Cu........................ = 23° 45′ 04,346″
φ_1Mo .... = 10° 41′ 22,94″
φ_1Cu — φ_1Mo = $\varkappa$ .... = 13° 03′ 41,406″
Directly measured $\varkappa$ .... = 13° 03′ 39,778″
Mean $\varkappa$ .... = 13° 03′ 40,592″

measurements with better adjustment of the crystal the difference between the maximum and the minimum of the three values obtained from the second set of measured data, is only 0,9 X. U. As it can be noted from the Table 2, the values derived from the glancing angles in both cases with bad and good crystal-adjustments greatly differ from one another, while the values of the grating constant derived from the measured values of the angle $\varkappa$ with bad and good adjustments of the crystal, which are 1908, 17 X. U. and 1908, 99 X. U. respectively, are not only at very little variance but also very nearly agree with one another. A comparison of both these values, obtained from $\varkappa$ measured with bad and good adjustments of the crystal, with one another and a comparison of the values of the grating constant derived from the values of the glancing angles φ, measured in both cases, with one another and also of these values with the values derived from the values of the angle $\varkappa$, reveal the special merit of the $\varkappa$-method.

We have chosen Cu $K\alpha_1$ and Mo $K\alpha_1$ because these lines[8]) are very precisely measured, and the angle $\varkappa$ (Cu $K\alpha_1$ — Mo $K\alpha_1$) may be taken to be nearly equal to the glancing angle φ of Mo $K\alpha_1$. It has been shown by the former authors that if the angle $\varkappa$ is smaller than the glancing angle φ, the defect of the crystal has a smaller influence on the value of the grating constant than if the angle $\varkappa$ is nearly equal to the glancing angle φ. But the errors of the scale in this case, as we have mentioned before, have again greater influence upon the results. To avoid this influence, we have taken a larger angle $\varkappa$ (even slightly larger than the glancing angle φ of Mo $K\alpha_1$).

So, from the above considerations, we see that through the $\varkappa$-method, the discrepancy due to the defects of the grating crystal is diminished, but is not altogether eliminated. Now we show that by the combination of the φ- and the $\varkappa$-methods, we can exactly determine the amount of possible displacement and its consequent error that enters in the value of the grating constant and thus practically eliminate all such errors due to the defects of the crystal even if they are great, the only condition naturally being that the said defects remain stationary during the course of observation. We mention this fact because the result calculated from the angle $\varkappa$ derived from the values of two different glancing angles measured with two different crystal-adjustments cannot be deemed to be more accurate than the results derived from the glancing angles.

Now to show the principle of the mentioned possibility of the combination of φ- and $\varkappa$-methods for the elimination of the discre-

[8]) A. Larson, Phil. Mag. (7) **3** (1927), 1136; Ina Wennerlöf, Ark. Mat., Astr. och Fysik. (A) **22** (1930), Nr. 8.

pancy due to the defects of the grating crystal, we discuss from our Table 4, the final readings compiled from our Tables 1 and 3 and their recpective results. In the Table 4 we have two different values, firstly that of the φ of Cu $K\alpha_1$, namely 23° 40' 11,9″ which gives us a value of 1914, 77 X. U., and secondly that of the glancing angle φ of Mo $K\alpha_1$, which gives us a value of 1923, 5 X. U. as the

Table 4.

Representation of the Ist series of measurements and their results ($\Delta \sim 0{,}401$ mm).

$\frac{1}{2}\delta_1 = 0°\ 05'\ 13{,}11''$

φ_1Cu meas. 23° 40′ 11,9″	$d_1 =$ 1914,77 XU	φ_1Cu $\sim$ to 1908,166 XU 23° 45′ 25,01″	
mean $\varkappa$ 13° 04′ 02,84″	$d_1 =$ 1908,166 XU	cor. $\varkappa$ 13° 03′ 40,77″	cor. $d_1 =$ 1908,978 XU
φ_1Mo meas. 10° 36′ 08,6″	$d_1 =$ 1923,53 XU	φ_1Mo $\sim$ to 1908,166 XU 10° 41′ 19,6″	

$\frac{1}{2}\delta_1 = 0°\ 05'\ 11''$

Δ mm	mean Δ mm	$\frac{1}{2}\delta$	$\varepsilon\delta\varkappa = -(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$
Cu 0,416	0,401	5′ 01,45″	— 22,07″
Mo 0,385		5′ 23,52″	

Representation of the II nd series of measurements and their results ($\Delta \sim 0{,}025$ mm).

$\frac{1}{2}\delta = 18{,}18''$

φ_1Cu meas. 23° 45′ 04,35″	$d_1 =$ 1908,6 XU	φ_1Cu $\sim$ to 1908,985 XU 23° 44′ 46,18″	
mean $\varkappa$ 13° 03′40,59″	$d_1 =$ 1908,985 XU	cor. $\varkappa$ 13° 03′ 41,947″	cor. $d_1 =$ 1908,935 XU
φ_1 Mo meas. 10° 41′22,94″	$d_1 =$ 1908,0 XU	φ_1Mo $\sim$ to 1908,985 XU 10° 41′ 02,90″	

$\frac{1}{2}\delta = 20{,}04''$

Δ mm	mean Δ mm	$\frac{1}{2}\delta$	$\varepsilon\delta\varkappa = -(\frac{1}{2}\delta\text{Cu} - \frac{1}{2}\delta\text{Mo})$
Cu 0,0242	0,025	18,417″	+ 1,36″
Mo 0,0248		19,772″	

difference 0,043 XU d'_1s cor.mean value = 1908,956 XU

constant of crystal grating. We have already pointed out that, in this case, the difference between these two values of the constant of crystal grating is only due to the displacement of the crystal and that the value $d_1 = 1908,\ 166$ X. U. derived from the value of the angle $\varkappa = 13°\ 4'\ 2{,}84''$, is very near to the correct value, in as much as the displacement and similar errors are greatly diminished in the values derived from it. Assuming this value of the grating constant to be the correct one, we have derived from it the corresponding values of the glancing angle φ of Cu Kα_1 and similarly that of Mo Kα_1. Thus we have the measured value of φ Cu Kα_1 and the calculated value of φ Cu Kα_1, corresponding to the value of $d_1 = 1908, 166$ X. U. and also similarly the measured and the calculated values of φ of Mo Kα_1. Since the calculated values of the glancing angles of Cu Kα_1 and Mo Kα_1 are derived from the approximately correct value of the grating constant, they can be taken to be approximately correct. The difference between the measured value and the calculated value which is assumed to be correct, gives half of the approximate displacement error that exists in the measured values of the glancing angles φ because $\varphi_{\text{cor}} = \varphi_{\text{mes}} \pm \pm \frac{1}{2}\delta$, where φ_{cor} is the value of the correct glancing angle, φ_{mes} is the value of the measured glancing angle and δ is the value of the displacement. We have therefore $\frac{1}{2}\delta = 5' \, 13{,}11''$ in the case of φ of Cu Kα_1 and $\frac{1}{2}\delta = 5' \, 11''$ in the case of φ of Mo Kα_1 which can be seen from the Table 4. They are approximately equal. In fact these displacement errors should be dissimilar, because any particular amount of displacement of the crystal Δ with two glancing angles such as those of Cu Kα_1, and Mo Kα_1 makes the displacement errors in the glancing angles different. Our values of $\frac{1}{2}\delta$, as can be noted from the above mentioned Table, are approximately equal, because the corresponding calculated values of the glancing angles in either case are not derived from the correct value of the grating constant of the crystal but are only obtained from a value very near to the true value. Or in other words, they are derived from a value of the angle $\varkappa$ having a minute error $\varepsilon\delta\varkappa$. From these values of these displacement errors of δ of Cu Kα_1 and δ of Mo Kα_1 we have calculated the value of the displacement of the crystal Δ by the above mentioned equation. We have thus obtained two values, the mean of which being 0,401 mm and this value can be taken as the value of the displacement of the crystal. From this value of the displacement of the crystal we can derive the correct values of the δ of $\varphi_{\text{Cu K}\alpha_1}$ and δ of $\varphi_{\text{Mo K}\alpha_1}$. From these data we can also derive the displacement error $\varepsilon\delta\varkappa$ of the measured angle $\varkappa$, and thus correct it by elimination from the following considerations. We have pointed out that in our case $\varkappa = \varphi_\nu - \varphi_\mu$ where φ_ν is the glancing angle of the wave-length λ_ν and φ_μ is the glancing angle of the wave-length λ_μ, when $\varphi_\nu > \varphi_\mu$. It has been already pointed out that $\varphi_{\text{cor}} = \varphi_{\text{mes}} \pm \frac{1}{2}\delta_\varphi$ where φ_{cor} is the correct value, φ_{mes} the measured value of the glancing angle and δ_φ is the value of the displacement of the particular measured glancing angle φ. The displacement error δ is added if the crystal is displaced away from the slit and subtracted if it is displaced towards the slit. So we have

$$\begin{aligned}
&\text{from}\ldots\ \varphi_{\nu\,\mathrm{cor}} = \varphi_{\nu\,\mathrm{mes}} \pm \tfrac{1}{2}\delta_{\varphi\nu}\\
&\text{and}\ldots\ \varphi_{\mu\mathrm{cor}} = \varphi_{\mu\mathrm{mes}} \pm \tfrac{1}{2}\delta_{\varphi\mu},\\
&\text{for angle } \varkappa\ldots\ \varkappa_{\mathrm{cor}} = \varphi_{\nu\,\mathrm{cor}} - \varphi_{\mu\mathrm{cor}},\\
&\text{and}\ldots\ \varkappa_{\mathrm{mes}} = \varphi_{\nu\mathrm{mes}} - \varphi_{\mu\mathrm{mes}}.
\end{aligned}$$

From these we can deduce an equation expressing the relation between the correct angle $\varkappa_{\mathrm{cor}}$, the measured angle $\varkappa_{\mathrm{mes}}$ and half the displacement errors $\frac{1}{2}\delta_{\varphi\nu}$ of the gancing angle φ_ν and $\frac{1}{2}\delta_{\varphi\mu}$ of the glancing angle φ_μ in the following way:

$$\begin{aligned}
\varkappa_{\mathrm{cor}} &= (\varphi_{\nu\,\mathrm{mes}} \pm \tfrac{1}{2}\delta_{\varphi\nu}) - (\varphi_{\mu\,\mathrm{mes}} \pm \delta_{\varphi\mu})\\
\varkappa_{\mathrm{cor}} &= (\varphi_\nu \pm \tfrac{1}{2}\delta_{\varphi\nu}) - (\varphi_\mu \pm \tfrac{1}{2}\delta_{\varphi\mu}),\\
&= \varkappa_{\mathrm{mes}} \pm \tfrac{1}{2}(\delta_{\varphi\nu} - \delta_{\varphi\mu}) \text{ or } \varkappa_{\mathrm{mes}} \pm (\tfrac{1}{2}\delta_{\varphi\nu} - \tfrac{1}{2}\delta_{\varphi\mu})\\
&= \varkappa_{\mathrm{mes}} \pm \varepsilon\delta\varkappa,
\end{aligned}$$

where

$$\varepsilon\delta\varkappa = \tfrac{1}{2}\,(\delta_{\varphi\nu} - \delta_{\varphi\mu}).$$

In our case

$$\varkappa_{\mathrm{cor}} = \varkappa_{\mathrm{mes}} \pm \tfrac{1}{2}(\delta\varphi_{\mathrm{Cu\,K}\alpha_1} - \delta\varphi_{\mathrm{Mo\,K}\alpha_1}).$$

and in this special case,

$$\begin{aligned}
\varkappa_{\mathrm{cor}} &= \varkappa + (5'\,1{,}45'' - 5'\,23{,}52'')\\
&= 13^\circ\,4'\,2{,}84'' - 0^\circ\,0'\,22{,}07''\\
&= 13^\circ\,3'\,40{,}77''.
\end{aligned}$$

We have thus through the combination of the φ- and $\varkappa$-methods obtained the corrected value of the angle $\varkappa$ which gives us the value of $d = 1908,\ 98$ X. U. This corrected value, as it can be seen from the Table 4, agrees very well with the value of $d_1 = 1908,\ 99$ X. U. obtained from $\varkappa$ measured in the second set of observations, with a better crystal-adjustement (Table 4). As it has been shown, there has been actually a displacement of 0,401 mm in the position of the crystal with which we have taken the first set of measurements shown already, and the great divergence in the values of the constant of crystal grating, derived from φ of Cu $K\alpha_1$ and that from φ of Mo $K\alpha_1$ has, been due to this displacement.

To verify all these resultats we have shifted away the crystal from the displaced position to 0,401 mm, there by improving the crystal-adjustment. Again we have taken a set of three series of measurements (one of the φ of Cu $K\alpha_1$, the second of the φ of Mo $K\alpha_1$ and the third of the angle $\varkappa = \varphi_{\mathrm{Cu\,K}\alpha_1} - \varphi_{\mathrm{Mo\,K}\alpha_1}$, which we have called the measurements with better crystal-adjustment and which can be seen in the Table 3 *a*, *b*, *c* and *d*. It can be further seen in the Table 4, that the great divergence in the results of the grating constant obtained from φ of Mo $K\alpha_1$ and from φ of Cu $K\alpha_1$, that we note in the first set of measurements, is in the second set of measurements very greatly diminished. Through the diminution of the said divergence in the value of the grating constant, the correctness of the calculated displacement of the crystal, with which the first set of measurements have been taken, is verified. As a survey of our direct experimental verification we wish to point out to the table No. 5 wherein only the measured values of the glancing angles of Cu $K\alpha$, and Mo $K\alpha_1$ of the Ist set (of measurements) and those of the IInd set of measurements are recorded. The difference between the values of the measured glancing angles of Cu $K\alpha_1$ and Mo $K\alpha_1$ of the Ist set of measurements and those of the IInd set of measurements gives the $^1/_2\delta\varphi_{Cu}$ and the $^1/_2\delta\varphi_{Mo}$. Thus $^1/_2\delta\varphi_{Cu} - {}^1/_2\delta\varphi_{Mo} = \varepsilon\delta\varkappa = 21{,}9''$ and this agrees fairly well with the calculated error $\varepsilon\delta\varkappa$ of the Ist set of measurements, which is about 22″, thereby establishing the validity of our method of determining the displacement of the reflecting surface of the crystal and of eliminating the error that enters in the value of the angles $\varkappa$ and consequently in the value of the grating constant of the crystal.

Table 5.

Measurements of the Ist and the IInd series.

	φ_1 measured I	φ_1 measured II	$\frac{1}{2}\delta$	$\varepsilon\delta\varkappa$
Cu	23° 40′ 11,9″	23° 45′ 04,35″	4′52,4″	— 21,9″
Mo	10° 36′ 08,6″	10° 41′ 22,94″	5′14,3″	

If we note the second set of measurements and similarly apply again our method we see that there still remains a displacement of 0,025 mm in as much as we could not bring the crystal to the calculated position as the regulating screw of the crystal table is not sufficiently precise for this purpose. Following the same process mentioned before, we have determined the $\varepsilon\delta\varkappa$ which is about $+ 1{,}3''$ and then eliminated this error from $\varkappa$. Thus we have again the corrected $\varkappa$ which gives 1908,935 X. U. as the final corrected value of the constant of crystal grating. The value of d_1, obtained merely from the measured $\varkappa$ without being corrected of the second set of readings, differs from the corrected value only by 0,05 X. U. The values of d_1 obtained from φ of Cu $K\alpha_1$ and φ of Mo $K\alpha_1$ measured with this crystal-adjustment differs from the correct value by 0,3 X. U. in the case of Cu and by 0,9 X. U. in the case of Mo. From this it is evident that if we calculate with this method, there is no need of further improvement of the crystal adjustment. Also if we compare the final corrected value of d_1 derived from the second set of readings with the corrected value of the constant