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ON A NONLINEAR TWO-POINT BOUNDARY VALUE PROBLEM

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1. Introduction

Boundary value problems for nonlinear differential systems of the second order have been in the centre of interest for a long time and were studied in many research works. One of the works to be mentioned here is the one performed by N. G. de Bruijn [1] in which the author offered an existence proof for the solution of a boundary value problem that arose in the work on gas discharges.

In the present paper the mentioned boundary value problem is studied in a more general setting and the existence of a solution to such generalized problem is established. It is worth noticing that boundary value problems of a similar type are studied in papers [2], [3].

Suppose that $a \geqq 0$, $b \geqq 0$, $\eta_0 > 0$, $\tau \geqq t_0 > 0$ are given real numbers. The following notation is used:

$$R = (-\infty, \infty),\ R_+ = [0, \infty),\ \alpha = \frac{1}{2}(a-1),\ \mathrm{J}_\alpha = \mathrm{J}_\alpha(\mathrm{x})$$

denotes the Bessel function of the first kind of order α and $\gamma = \gamma(\alpha)$ denotes the first positive zero of this function; $C^n(I)$ denotes the space of all functions which are n times continuously differentiable on an interval I, $C^0(I)$ means continuity.

The pair (y_1, y_2) of functions $y_1, y_2 \in C^0([0, t_0]) \cap C^2((0, t_0])$

has to satisfy

$$\begin{aligned} -\frac{d^2y_1}{dt^2} - \frac{a}{t}\frac{dy_1}{dt} &= y_1 H_1\left(t, y_1, y_2, \frac{dy_1}{dt}, \frac{dy_2}{dt}\right), \\ \frac{d^2y_2}{dt^2} + \frac{b}{t}\frac{dy_2}{dt} &= y_2 H_2\left(t, y_1, y_2, \frac{dy_1}{dt}, \frac{dy_2}{dt}\right), \end{aligned} \tag{I}$$

$$y_1(t) > 0 \ (0 \leqq t < t_0), \ y_1(t_0) = 0, \quad \frac{dy_1}{dt}(0) = 0,$$
$$y_2(0) > 0, \qquad y_2(t_0) = \eta_0, \quad \frac{dy_2}{dt}(0) = 0, \tag{II}$$

where the following conditions are always assumed:

($\mathbf{A}_1$) H_i: $[0, \tau] \times R^4 \to R$ $(i = 1,2)$ are continuous

($\mathbf{A}_2$) for each compact set $[0, \tau]\, xD$ where $D \subset R^4$ there exists a constant $L > 0$ such that

$$|z_i H_i(t, z_1, z_2, z_3, z_4) - \bar{z}_i H_i(t, \bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4)| \leqq L \sum_{j=1}^{4} |z_j - \bar{z}_j|$$

$(i = 1, 2)$

for any pairs of points (t, z_1, z_2, z_3, z_4), $(t, \bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4) \in [0, \tau]\, xD$.

($\mathbf{A}_3$) there exist continuous increasing and upper unbounded functions $H_j^{(i)}$: $R_+ \to R_+$ $(i, j = 1, 2)$ with $H_1^{(1)}(0) = H_2^{(1)}(0) \geqq \gamma^2 \tau^{-2}$, $H_1^{(2)}(0) = H_2^{(2)}(0) = 0$ such that the inequalities

$$H_1^{(1)}(z_2) \leqq H_1(t, z_1, z_2, z_3, z_4) \leqq H_2^{(1)}(z_2),$$
$$H_1^{(2)}(z_1) \leqq H_2(t, z_1, z_2, z_3, z_4) \leqq H_2^{(2)}(z_1)$$

are fulfilled on the set $[0, \tau] \times R_+^2 \times R^2$.

By the transformation $t = A^{-\frac{1}{2}}x$, where $A = H_1^{(1)}(0) = H_2^{(1)}(0)$ and

$$y_1(t) = u(x), \ y_2(t) = v(x), \ A^{\frac{1}{2}}\tau = \tau_0, \ A^{\frac{1}{2}}t_0 = \xi_0, \ \frac{d}{dx} = {}',$$
$$A^{-1}H_1(A^{-\frac{1}{2}}x, z_1, z_2, A^{\frac{1}{2}}z_3, A^{\frac{1}{2}}z_4) = f(x, z_1, z_2, z_3, z_4),$$
$$A^{-1}H_2(A^{-\frac{1}{2}}x, z_1, z_2, A^{\frac{1}{2}}z_3, A^{\frac{1}{2}}z_4) = g(x, z_1, z_2, z_3, z_4),$$
$$A^{-1}H_i^{(1)}(z) - 1 = f_i(z), \ A^{-1}H_i^{(2)}(z) = g_i(z) \ (i = 1, 2),$$

we get

$$u'' + ax^{-1}u' + f(x, u, v, u', v')\, u = 0, \tag{1.1}$$

$$v'' + bx^{-1}v' - g(x, u, v, u', v')\, v = 0, \tag{1.2}$$

$$u(x) > 0 \ (0 \leqq x < \xi_0), \ u(\xi_0) = 0, \quad u'(0) = 0,$$
$$v(0) > 0, \qquad v(\xi_0) = \eta_0, \ v'(0) = 0. \tag{1.3}$$

It is evident that properties of the functions H_1, H_2 given in assumptions ($\mathbf{A}_1$), ($\mathbf{A}_2$) remain valid also for the functions f, g (except for interval $[0, \tau]$ that is substituted by $[0, \tau_0]$). The assumption ($\mathbf{A}_3$) of f, g functions assumes the form

($\mathbf{A}_3'$) inequalities

$$1 + f_1(z_2) \leqq f(x, z_1, z_2, z_3, z_4) \leqq 1 + f_2(z_2), \tag{1.4}$$

$$g_1(z_1) \leqq g(x, z_1, z_2, z_3, z_4) \leqq g_2(z_1) \tag{1.5}$$

hold on the set $[0, \tau_0] \times R_+^2 \times R^2$ where f_i, g_i: $R_+ \to R$ are continuous increasing and upper unbounded functions, $f_i(0) = g_i(0) = 0$ $(i = 1, 2)$.

Finally, let F_i and G_i denote the inverse function of f_i and of g_i, respectively $(i = 1, 2)$.

For general positive values of ξ_0 and η_0 the sufficient (neccesary) condition for existence of a solution of the boundary value problem is

$$\gamma[1 + f_1(\eta_0)]^{-\frac{1}{2}} < \xi_0 < \gamma \qquad (\gamma[1 + f_2(\eta_0)]^{-\frac{1}{2}} < \xi_0 < \gamma)$$

which reduces to

$$H_1^{(1)}(0) < \left(\frac{\gamma}{t_0}\right)^2 < H_1^{(1)}(\eta_0) \qquad \left(H_2^{(1)}(0) < \left(\frac{\gamma}{t_0}\right)^2 < H_2^{(1)}(\eta_0)\right).$$

2. The topological method

The solution of the given boundary value problem can be obtained N. G. de Bruijnś method used in [1]:

Let p and q be real numbers, $p > 0$, $q \geqq 0$. We observe the solution of the system (1.1), (1.2) with initial values

$$u(0) = p, \ v(0) = q, \ u'(0) = 0, \ v'(0) = 0. \tag{2.1}$$

G. P. Grizans, J. A. Klokov [4] proved a theorem on the existence of a solution of certain initial value problems. We formulate the theorem in a special case.

Theorem A ([4, p. 24]). Let $x_0 > 0$, $k_i > -1$, y_{oi} $(i = 1, 2)$ be real numbers and let the functions h_i: $[0, x_0] \times R^4 \to R$ $(i = 1, 2)$ be continuous. Then the problem

$$y_i'' + \frac{k_i}{x} y_i' = h_i(x, \boldsymbol{y}, \boldsymbol{y}')$$

$$y_i(0) = y_{oi},\ y_i'(0) = 0,\ i = 1, 2,$$

where $\boldsymbol{y} = (y_1, y_2)$, has a solution $\boldsymbol{y} \in C([0, \bar{x}_0]) \cap C^2((0, \bar{x}_0])$ for some $\bar{x}_0 \in (0, x_0)$. Moreover, if the functions h_i $(i = 1, 2)$ satisfies Lipschitz condition on a neighbourhood $U(\boldsymbol{r})$ of the point $\boldsymbol{r} = (0, y_1(0), y_2(0), 0,0)$, i.e. there exists a constant $L > 0$ such that

$$|h_i(x, z_1, z_2, z_3, z_4) - h_i(x, \bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4)| \leqq L \sum_{j=1}^{4} |z_j - \bar{z}_j|,$$

$i = 1, 2$ for any pairs of points (x, z_1, z_2, z_3, z_4), $(x, \bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4) \in U(\boldsymbol{r})$, then the problem has exactly one solution and moreover this solution continuously depends on the initial conditions.

Clearly the assumptions of *Theorem A* are satisfied by conditions (A_1) and (A_2). From *Theorem A* it follows that the problem (1.1), (1.2), (2.1) has exactly one solution on some interval $[0, \bar{\tau}_0]$ $(0 < \bar{\tau}_0 \leqq \tau_0)$ and moreover this solution continuously depends on the variables p, q.

For x increasing from 0 onwards as long as $u > 0$ by (1.5) we get $(x^b v')' > 0$ and then v increases (except for the case that v is identically zero), and u decreases while $u > 0$, $v \geqq 0$. Since in this case we have $(x^a u')' < 0$, so there is a finite ξ such that $u(x) = 0$ for the first time at $x = \xi$ (actually it follows from *Lemma 2* (see Sec. 3) that

$$\xi \leqq \gamma[1 + f_1(q)]^{-\frac{1}{2}}.$$

We define η by $\eta = v(\xi)$. The numbers ξ and η (cf. *Theorem A*) are uniquely determined by p and q (precisely: the point (ξ, η) is uniquely determined by the point $(u(0), v(0), u'(0), v'(0)) = (p, q, 0, 0))$, and the region $p > 0$, $q \geqq 0$ is mapped continuously into the region $\xi > 0$, $\eta \geqq 0$.

Let us denote this continuous mapping by $\boldsymbol{\Phi}$. This $\boldsymbol{\Phi}$ is vector-valued:

$$\boldsymbol{\Phi}(p, q) = (\xi, \eta).$$

The point $(\gamma, 0)$ clearly belongs to the range of $\boldsymbol{\Phi}$. Indeed, for every $p > 0$ and $q = 0$ the pair (u_0, v_0) of the functions $u_0(x) = p\Gamma(\alpha + 1)2^\alpha x^{-\alpha} J_\alpha(x)$, $v_0(x) \equiv 0$ $(0 \leqq x \leqq \gamma)$ (where $\alpha = \frac{1}{2}(a - 1)$, Γ is the gamma function, $J_\alpha = J_\alpha(x)$ is the Bessel function and γ is the first positive zero of this function) satisfies (1.1), (1.2) and $u_0(x) \to p$ for $x \to 0^+$, $v(0) = 0$. Since γ is the first positive zero of the function u_0 then $\eta = v_0(\gamma) = 0$ i.e. $\Phi\,(p, 0) = (\gamma, 0)$ for every $p > 0$. For all other solutions we have $\eta > 0$. Let ξ_0, η_0 be positive numbers satisfying

$$\gamma[1 + f_1(\eta_0)]^{-\frac{1}{2}} < \xi_0 < \gamma.$$

We shall show that (ξ_0, η_0) belongs to the range of Φ by means of the winding method. We produce a closed curve in the (p, q) — domain and show that its image under Φ is a curve encircling the point (ξ_0, η_0). It follows that the boundary value problem (1.1), (1.2), (1.3) has at least one solution. The contour in the (p, q) — plane will be the rectangle $D = \{(p, q) \in R^2\colon \sigma < p < P, 0 < q < \eta_1\}$ shown in *Fig. 1*. The number η_1 can be any number greater than η_0.

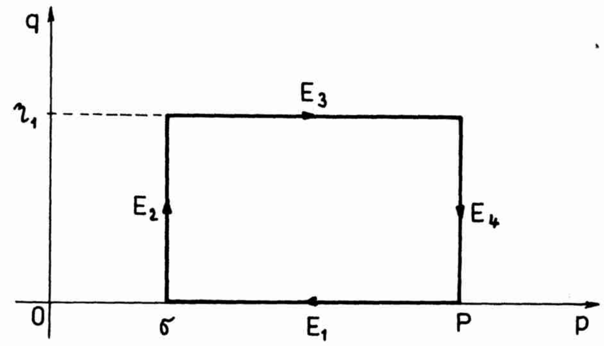

Fig. 1

The number σ has to be small, P has to be large. The image of $E_1 = \{(p, q) \in R^2\colon \sigma \leqq p \leqq P,\ q = 0\}$, $E_2 = \{(p, q) \in R^2\colon p = \sigma,\ 0 \leqq q \leqq \eta_1\}$, $E_3 = \{(p, q) \in R^2\colon \sigma \leqq p \leqq P, q = \eta_1\}$, $E_4 = \{(p, q) \in R^2\colon p = P, 0 \leqq q \leqq \eta_1\}$ in the (ξ, η) — plane is shown in *Fig. 2*. Note that if $(p, q) \overset{\Phi}{\mapsto} (\xi, \eta)$ then a pair (u, v) of functions u, v satisfies (1.1), (1.2) and initial conditions $u(0) = p, v(0) = q, u'(0) = 0, v'(0) = 0$ has the property: $u(x) > 0$ $(0 \leqq x < \xi)$, $u(\xi) = 0$, $v(\xi) = \eta$. The image of E_1 is the single point $(\gamma, 0)$. The image of E_2 closely resembles the part of the curve $\eta = F_1(\gamma^2\xi^{-2} - 1)$, at least σ is small. The image of E_3 is safely above the line $\eta = \eta_1$. The real difficulty lies in studying the image of E_4. If ξ_1 is any number, $\xi_1 < \gamma$ (if $\xi_0 < \gamma$ we can take care that $\xi_0 < \xi_1 < \gamma$) we can show that P can by taken so large that the image of E_4 stays outside the rectangle with vertices $(0, 0)$, $(0, \eta_1)$, $(\xi_1, 0)$, (ξ_1, η_1).

In *Fig. 2* we have drawn (ξ_0, η_0) such that $0 < \xi_0 < \gamma$, $\eta_0 > F_1(\gamma^2\xi_0^{-2} - 1)$. From the situation shown in *Fig. 2* it follows that index $\underset{\Phi(\partial D)}{\operatorname{ind}}(\xi_0, \eta_0)$ of the point (ξ_0, η_0) relative to the curve $\Phi(\partial D)$, where $\partial D = E_1 \cup E_2 \cup E_3 \cup E_4$, is not zero.

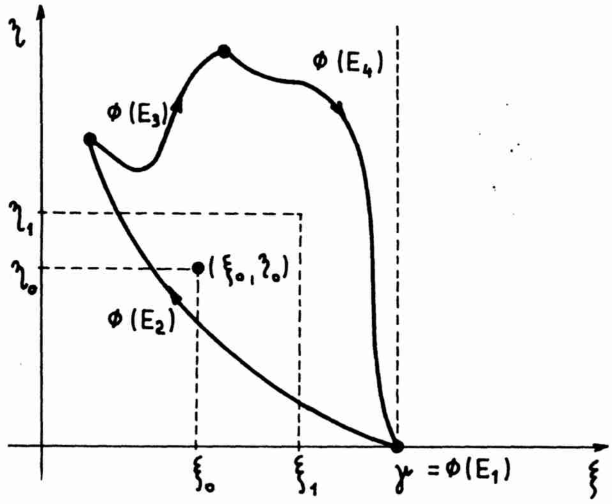

Fig. 2

Since D is an open and bounded set in R^2, the mapping $\mathbf{\Phi}$: $D \cup \partial D \to R^2$ is continuous and the point $(\xi_0, \eta_0) \notin \mathbf{\Phi}(\partial D)$, then there exists Brouwer degree $\deg(\mathbf{\Phi}, D, (\xi_0, \eta_0))$ of the mapping $\mathbf{\Phi}$ with respect to the set D and to the point (ξ_0, η_0). Moreover we have $deg(\mathbf{\Phi}, D, (\xi_0, \eta_0)) = \underset{\mathbf{\Phi}(\partial D)}{\text{ind}}(\xi_0, \eta_0)$. Consequently, $deg(\mathbf{\Phi}, D, (\xi_0, \eta_0)) \neq 0$. It follows that the point (ξ_0, η_0) belongs to the range of $\mathbf{\Phi}$ i.e. there exists a point (p_0, q_0) such that the pair (u, v) of functions u, v satisfies (1.1), (1.2) and the initial conditions $u(0) = p_0$, $v(0) = q_0$, $u'(0) = 0$, $v'(0) = 0$ have the property $u(x) > 0$ $(0 \leqq x < \xi_0)$, $u(\xi_0) = 0$, $v(0) > 0$, $v(\xi_0) = \eta_0$.

In Sec. 3 we show some lemmas needed for the final conclusion in Sec. 4.

3. Lemmas

Lemma 1. Let β, s, s_1 be real numbers, $0 < \beta < 1$, $0 < s < \tau_0$, $1 < ss_1^{-1} < 2$. We abbreviate

$$Q = [2(ss_1^{-1})^b + 4\, ln\, \beta^{-1}]^2 (s - s_1)^{-2}; Q_1 = G_1(Q). \tag{3.1}$$

Let $U \in C^1([0, s])$, $U(x) \geqq Q_1$ $(0 \leqq x \leqq s)$ and let v be a solution of the differential equation

$$v'' + bx^{-1}v' - g(x, U(x), v, U'(x), v')v = 0 \tag{3.2}$$

on the interval $[0, s]$ with $v'(0) = 0$, $v(0) > 0$. Then we have

$$0 < v(x) < \beta v(s) \qquad (0 \leqq x \leqq s_1). \tag{3.3}$$

Proof. The functions v and v' are positive throughout $0 \leqq x \leqq s$ (see the beginning of Sec. 2). Putting $\frac{v'}{v} = y$ by using (1.5) and from monotonicity of function g_1 we have

$$y' + y^2 + bx^{-1}y = g(x,\ U(x),\ v,\ U'(x),\ v') \geqq g_1(U(x)) \geqq g_1(Q_1),\ y \geqq 0.$$

Hence $g_1(Q_1) = g_1(G_1(Q)) = Q$ we have

$$y' + y^2 + bx^{-1}y \geqq Q, \qquad y(x) \geqq 0\ (0 \leqq x \leqq s).$$

If $y(s_0) = \frac{1}{2} Q^{\frac{1}{2}}$ for some point $s_0 \in [s_1, s]$ we have $y'(s_0) > 0$. Indeed, for this point it holds $y'(s_0) \geqq -\frac{b}{2s_1} Q^{\frac{1}{2}} + \frac{3}{4} Q > 0$. Therefore we can have $y \leqq \frac{1}{2} Q^{\frac{1}{2}}$ at most on an interval $s_1 \leqq x \leqq s_0$ with $s_0 \leqq s$. On that interval we have

$$(x^b y)' \geqq \frac{1}{2} x^b Q, \qquad 0 \leqq x^b y \leqq \frac{1}{2} x^b Q^{\frac{1}{2}}.$$

Hence the length of the interval cannot exceed $(s_2 s_1)^b Q^{-\frac{1}{2}}$. At this is at most $\frac{1}{2}(s - s_1)$, we have $y > \frac{1}{2} Q^{\frac{1}{2}}$ at least on $\frac{1}{2}(s + s_1) \leqq x \leqq s$. Hence

$$ln \frac{v(s)}{v(s_1)} = \int_{s_1}^{s} y(x)\, dx \geqq \frac{1}{2} Q^{\frac{1}{2}} \cdot \frac{1}{2}(s - s_1) > ln\, \beta^{-1}$$

and it follows that $v(s_1) < \beta v(s)$.

Then from monotonicity of function v we get

$$0 < v(x) < \beta v(s) \qquad (0 \leqq x \leqq s_1).$$

Lemma 2. Let h_1, h_2 be real numbers, $1 \leqq h_1 < h_2$. Let $w \in C^1([0, \gamma])$, $w(x) \geqq 0\ (0 \leqq x \leqq \gamma)$ and let for each $x \in [0, \gamma]$, $z_1 \in R_+$, $z_3 \in R$ hold the inequality

$$h_1 \leqq f(x,\ z_1,\ w(x),\ z_3,\ w'(x)) \leqq h_2. \tag{3.4}$$

Let u be a solution of the equation

$$u'' + ax^{-1}u' + f(x,\ u,\ w(x),\ u',\ w'(x))\, u = 0 \tag{3.5}$$

with $u(0) > 0$, $u'(0) = 0$. Then there is a number ξ with

$$\gamma h_2^{-\frac{1}{2}} \leqq \xi \leqq \gamma h_1^{-\frac{1}{2}} \tag{3.6}$$

such that u is positive for $0 \leqq x < \xi$ and zero at ξ.

Moreover, if x_0 is any number with $0 < x_0 < \xi$, and if $-\dfrac{u'(x_0)}{u(x_0)}$ is abbreviated to $r(x_0)$, then we have $r(x_0) > 0$ and

$$x_0 + [2h_2^{\frac{1}{2}} + 2r(x_0)]^{-1} \leqq \xi < x_0 + Kr^{-1}(x_0)\, h_2^{\frac{a^*}{2}}, \tag{3.7}$$

where $a^* = \max\{1; a\}$,

$$K = \max\left\{2^a; \gamma \max_{0 \leqq x \leqq \frac{1}{2}\gamma} \frac{\mathrm{J}_{a+1}(x)}{\mathrm{J}_a(x)}\right\}. \tag{3.8}$$

Proof. Putting $y = \dfrac{u'}{u}$ we get, instead of (3.5),

$$y' + y^2 + ax^{-1}y + f(x, u, w(x), u', w'(x)) = 0, \; y(0) = 0. \tag{3.9}$$

Consider the functions u_1, and u_2, defined by

$$u_i(x) = x^{-a} J_a(h_i x) \qquad (i = 1, 2).$$

It can be verified that the function y_i $(i = 1, 2)$ defined by

$$y_i(x) = \frac{u_i'(x)}{u_i(x)} \qquad (0 \leqq x < h_i^{-\frac{1}{2}}\gamma)$$

has the following properties:

$$y_i' + y_i^2 + ax^{-1}y_i + h_i = 0, \qquad y_i(0) = 0,$$

$$y_i(x) < 0 \qquad (0 < x < h_i^{-\frac{1}{2}}\gamma),\; y_i(x) \to -\infty \text{ for } x \to h_i^{-\frac{1}{2}}\gamma^-,$$

and

$$y_i(x) = -h_i^{\frac{1}{2}} \frac{J_{a+1}(h_i^{\frac{1}{2}}x)}{J_a(h_i^{\frac{1}{2}}x)} \qquad (0 \leqq x < h_i^{-\frac{1}{2}}\gamma). \tag{3.10}$$

For each point $(x, y_i(x))$, equation (3.9) defines the value of the derivative y', or the slope of the tangent line to the integral curve passing through this point (if such exist). Inquiring the line elements of the differential equation (3.9) on the curves y_1, y_2 we can easily find out that everywhere where the functions y_1, y, y_2 are defined it holds $y_2 \leqq y \leqq y_1$. Indeed, $y_1(0) = y_2(0) = y(0) = 0$ and

$$(y')_{(x,\,y_1(x))} = -y_1^2(x) - ax^{-1}y_1(x) - f(x,\, u_1(x),\, w(x),\, u_1'(x),\, w'(x)) \leqq$$
$$\leqq -y_1^2(x) - ax^{-1}y_1(x) - h_1 = y_1'(x),$$

$$(y')_{(x,\,y_2(x))} = -y_2^2(x) - ax^{-1}y_2(x) - f(x,\, u_2(x),\, w(x),\, u_2'(x),\, w'(x)) \geqq$$
$$\geqq -y_2^2(x) - ax^{-1}y_2(x) - h_2 = y_2'(x).$$

It follows that the solution of (3.9) stays between these two curves. Therefore it has a vertical asymptote $x = \xi$ with some ξ satisfying (3.6). The situation is shown in *Fig. 3*. It follows that $u(x) > 0$ on $0 \leqq x < \xi$ and $u(\xi) = 0$.

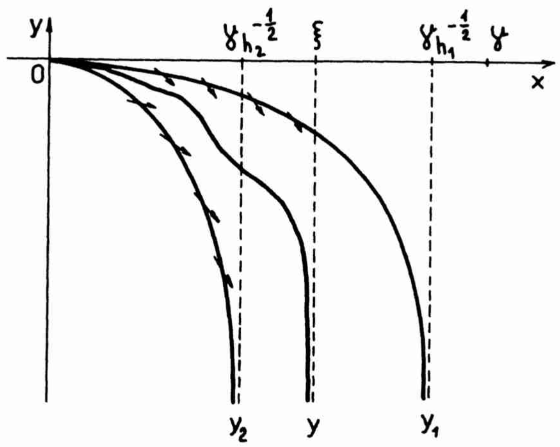

Fig. 3

Next we take an x_0 with $0 < x_0 < \xi$, we put $r(x_0) = -\dfrac{u'(x_0)}{u(x_0)}$ and we take a number s_0 with $s_0 > r(x_0)$. Through the point $(x_0,\, -s_0)$ we draw the curve

$$\tilde{y} = \frac{s_0(\lambda - x_0)}{x - \lambda},\quad x_0 \leqq x < \lambda \tag{3.11}$$

where $\lambda = x_0 + \dfrac{s_0}{s_0^2 + h_2}$. It is obvious that

$$\tilde{y}'(x) = -\tilde{y}^2(x) - h_2\left(\frac{\lambda - x_0}{x - \lambda}\right)^2 \leqq -\tilde{y}^2(x) - ax^{-1}\tilde{y}(x) - h_2 \leqq$$

$$\leqq -\tilde{y}^2(x) - ax^{-1}\tilde{y}(x) - f(x, \tilde{u}(x), w(x), \tilde{u}'(x), w'(x)) =$$

$$= (y')_{(x, \tilde{y}(x))}, \text{ where } \frac{\tilde{u}'(x)}{\tilde{u}(x)} = \tilde{y}(x),\ \tilde{u}(x) > 0\,(x_0 \leqq x < \lambda).$$

Hence the line elements of (3.9) cut the curve (3.11) as shown in Fig. 4, and we infer that our solution of (3.9), which passes through point $(x_0, -r(x_0))$, stays to the right of that curve. Hence $\lambda \leqq \xi$.

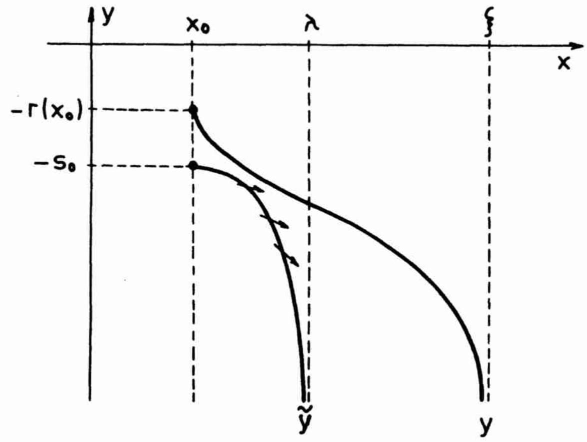

Fig. 4

Up to now we have shown that $\xi \geqq x_0 + \dfrac{s_0}{h_2 + s_0^2}$ for all $s_0 \geqq r(x_0)$. The function $\dfrac{s_0}{h_2 + s_0^2}$, considered as a function of s_0, has its maximum at $s_0 = h_2^{\frac{1}{2}}$, whence $\xi \geqq x_0 + \dfrac{1}{2} h_2^{-\frac{1}{2}}$ if $r(x_0) \leqq h_2^{\frac{1}{2}}$.

If $r(x_0) > h_2^{\frac{1}{2}}$ we still have $\xi \geqq x_0 + \dfrac{r(x_0)}{r^2(x_0) + h_2}$.

We easily infer that $\xi \geqq x_0 + [2h_2^{\frac{1}{2}} + 2r(x_0)]^{-1}$ holds in both cases. Indeed, it $0 < r(x_0) < h_2^{\frac{1}{2}}$ then $x_0 + \dfrac{1}{2} h_2^{-\frac{1}{2}} = x_0 + (2h_2^{\frac{1}{2}})^{-1} \geqq x_0 + [2h_2^{\frac{1}{2}} + 2r(x_0)]^{-1}$. For $r(x_0) > h_2^{\frac{1}{2}}$ we have $r(x_0) > \dfrac{h_2}{r(x_0)}$ and, consequently, $x_0 + \dfrac{r(x_0)}{r^2(x_0) + h_2} = x_0 +$

$$+ \left[r(x_0) + \frac{h_2}{r(x_0)}\right]^{-1} \geqq x_0 + (2r(x_0))^{-1} \geqq x_0 + [2h_2^{\frac{1}{2}} + 2r(x_0)]^{-1}.$$

In order to prove the upper estimate of (3.7) we transform the equation (3.9), putting $x^a y(x) = z(x)$ for $a > 0$:

$$z' + x^{-a} z^2 + x^a f(x, u(x), w(x), u'(x), w'(x)) = 0.$$

Consequently $z' + x^{-a} z^2 < 0$ for $0 < x < \xi$ (since $x^a f > 0$). It follows that $(z^{-1})' > x^{-a}$. As $z < 0$ for $x_0 \leqq x < \xi$, and $z^{-1} \to 0$ as $x \to \xi^-$, we infer that

$$(x_0^a r(x_0))^{-1} = \int_{x_0}^{\xi} (z^{-1})' \, dx \geqq \int_{x_0}^{\xi} x^{-a} \, dx > \int_{x_0}^{\xi} \xi^{-a} \, dx = (\xi - x_0)\, \xi^{-a},$$

hence $(\xi - x_0)\, r(x_0) < \left(\frac{\xi}{x_0}\right)^a$ for $a > 0$.

This inequality is useless if x_0 is small, but we know (cf. Fig. 3) that for $0 < x_0 < \frac{1}{2} \gamma h_2^{-\frac{1}{2}}$ we have $r(x_0) \leqq \left|\frac{u_2'(x_0)}{u_2(x_0)}\right|$.

Hence

$$(\xi - x_0)\, r(x_0) < \xi r(x_0) \leqq h_2^{\frac{1}{2}} \gamma \max_{0 \leqq x \leqq \frac{1}{2}\gamma} \left(\frac{J_{\alpha+1}(x)}{J_\alpha(x)}\right) \qquad \text{if } x_0 < \frac{1}{2} \gamma h_2^{-\frac{1}{2}}.$$

If $x_0 \geqq \frac{1}{2} \gamma h_2^{-\frac{1}{2}}$ we have $(\xi - x_0)\, r(x_0) < \left(\frac{\xi}{x_0}\right)^a < 2^a \gamma^{-\omega} h_2^{\frac{a}{2}} \xi^a$. So if we take K and a^* as in (3.8) and we use $\xi \leqq \gamma$, we get the upper estimate of (3.7) for $a > 0$. That upper estimate also holds for $a = 0$ because $(\xi - x_0)\, r(x_0) \leqq 1$ and $h_2 > 1$.

Lemma 3. Let ξ_1 and η_1 be real numbers, and assume $0 < \xi_1 < \gamma$, $\eta_1 > 0$. Then there is a positive number P with the following property:

If ξ is a real number with $0 < \xi < \xi_1$ and if the functions $u, v \in C^1([0, \xi])$ satisfy (1.1) and (1.2) for $0 < x \leqq \xi$ with $u(0) \geqq P$, $u(x) > 0$ $(0 \leqq x < \xi)$, $u(\xi) = 0$, $v(0) > 0$, $u'(0) = v'(0) = 0$, then we have

$$v(\xi) > \eta_1.$$

Proof. On the interval $0 \leqq x \leqq \xi_1$ the function $u_0(x) = u(0)\, \Gamma(\alpha + 1)\, 2^\alpha x^{-\alpha} \cdot J_\alpha(x)$ (where Γ is the gamma function, $\alpha = \frac{1}{2}(a - 1)$, $J_\alpha(x)$ is the Bessel function) satisfies

$$u_0'' + a x^{-1} u_0' + u_0 = 0$$

and has initial values $u_0(0) = u(0)$ (precisely $u_0(x) \to u(0)$ for $x \to 0^+$), $u_0'(0) = u'(0) = 0$.

Since the functions u, v satisfy

$$u'' + ax^{-1}u' + f(x, u, v, u', v')u = 0$$

on the interval $(0, \xi]$, it is obvious that functions

$$y = \frac{u'}{u}, \qquad y_0 = \frac{u_0'}{u_0} \qquad \text{satisfy}$$

$$y' + y^2 + ax^{-1}y + [f(x, u, v, u', v') - 1] + 1 = 0, \qquad y(0) = 0, \tag{3.12}$$

$$y_0' + y_0^2 + ax^{-1}y_0 + 1 = 0, \qquad y_0(0) = 0.$$

Let M be the maximum of $\left|\frac{u_0'(x)}{u_0(x)}\right| = \frac{J_{a+1}(x)}{J_a(x)}$ for $0 \leqq x \leqq \xi_1$. If the "perturbation" $f(x, u, v, u', v') - 1$ in (3.12) is "small", the value of $y(x)$ does not deviate much from $y_0(x)$. Note that this fact follows from the continuous dependence of the solutions (3.12) on parameters. Namely, the equation satisfies uniqueness condition and its "right — hand side'" is continuous for a solution y with condition $y(0) = 0$. From (1.4) and from properties of the functions f_1, f_2 it follows that $f(x, u, v, u', v') - 1$ is to be "small" under the condition v be "small".

In particular we can find a number $\delta > 0$ with $0 < \delta < \eta_1$ such that for all p with $0 \leqq p \leqq \xi$ the following is true:

If $|v(x)| < \delta$ for $0 \leqq x \leqq p$ then

$$\left|\frac{u'(x)}{u(x)}\right| \leqq 2M \qquad (0 \leqq x \leqq p).$$

We take a^*, K as in (3.8) and we define

$$\beta = \frac{\delta}{\eta_1},$$

$$\mu = [(4 + 3\gamma^{-1})(1 + f_2(\eta_1)^{\frac{1}{2}} + 8M]^{-1},$$

$$P = G_1\{\mu^{-2}[2(\gamma\mu^{-1})^b + 4\ln\beta^{-1}]^2\}\exp[K\gamma\mu^{-1}(1 + f_2(\eta_1))^{\frac{a^*}{2}}],$$

$$s_1 = \xi - 2\mu,$$

$$s = \xi - \mu.$$

Note that δ, β, μ, M and P depend only on ξ_1 and η_1. The numbers s_1 and s, however, depend on ξ_1, η_1 and ξ. We shall assume that $u(0) \geqq P$, $0 < \xi < \xi_1$, $v(\xi) \leqq \eta_1$ and we derive a contradiction.

As v is increasing for $0 \leqq x \leqq \xi$ (see Sec. 2) we have

$$0 < v(0) \leqq v(x) \leqq v(\xi) \leqq \eta_1 \qquad (0 \leqq x \leqq \xi).$$

From (1.4) and from monotonicity of functions f_1, f_2 we get

$$1 + f_1(v(0)) \leqq f(x, u(x), v(x), u'(x), v'(x)) \leqq 1 + f_2(v(x)) \leqq 1 + f_2(\eta_1) \tag{3.13}$$

for $0 \leqq x \leqq \xi$.

We define functions U, $\bar{U}$, V, $\bar{V}$ on the interval $[0, \gamma]$:

$$U(x) = u(x),\ \bar{U}(x) = u'(x),\ V(x) = v(x),\ \bar{V}(x) = v'(x)\,(0 \leqq x \leqq \xi),$$

$$U(x) = u(\xi),\ \bar{U}(x) = u'(\xi),\ V(x) = v(\xi),\ \bar{V}(x) = v'(\xi)\,(\xi < x \leqq \gamma).$$

Consider the initial value problem

$$z'' + ax^{-1}z' + f(x, U(x), V(x), \bar{U}(x), \bar{V}(x))\,z = 0, \tag{3.14}$$

$$z(0) = u(0),\ z'(0) = u'(0) = 0.$$

This problem has exactly one solution and obviously

$$z(x) = u(x) \text{ for } 0 \leqq x \leqq \xi \text{ (see beginning of Sec. 2).}$$

Then $z(x) > 0$ $(0 \leqq x < \xi)$, $z(\xi) = 0$, $z'(0) = 0$.
Hence by (3.13)

$$1 + f_1(v(0)) \leqq f(x, U(x), V(x), \bar{U}(x), \bar{V}(x)) \leqq 1 + f_2(\eta_1) \qquad (0 \leqq x \leqq \gamma).$$

Lemma 2 can be applied to the equation (3.14).
The property that z is positive for $0 \leqq x < \xi$ and zero at ξ, determines ξ uniquely, and therefore the ξ in Lemma 2 is the same as the one we have here. In particular (3.6) says that

$$\gamma[1 + f_2(\eta_1)]^{-\frac{1}{2}} \leqq \xi \text{ (and also } \xi \leqq \gamma[1 + f_1(v(0)]^{-\frac{1}{2}}).$$

It follows that $\xi > 3\mu$ and therefore

$$0 < \mu < s_1 < s < 2s_1.$$

By (3.7) we have

$$r(x) = -\frac{u'(x)}{u(x)} < [1 + f_2(\eta_1)]^{\frac{a^*}{2}} \frac{K}{\xi - x} \qquad (0 \leqq x \leqq s)$$

(for $x = 0$ we have $r(x) = 0$) and it follows by integration that

$$\ln \frac{u(0)}{u(s)} \leqq K[1 + f_2(\eta_1)]^{\frac{a^*}{2}} \int_0^s \frac{dx}{\xi - x} \leqq K[1 + f_2(\eta_1)]^{\frac{a^*}{2}} \frac{s}{\mu} \qquad (0 \leqq x \leqq s).$$

Then from the monotonicity of the function u we obtain

$$u(x) \geqq u(0) \exp\{-Ks\mu^{-1}[1 + f_2(\eta_1)]^{\frac{a^*}{2}}\} \qquad (0 \leqq x \leqq s).$$

Since $u(0) \geqq P$, we derive $u(x) \geqq G_1(Q) = Q_1$ $(0 \leqq x \leqq s)$ with Q given by (3.1) (note that $s < \gamma$, $0 < \mu < s_1 < s < 2s_1$, $\mu = s - s_1$, $0 < \beta < 1$). By Lemma 1 it holds

$$0 < v(x) < \beta v(s) \qquad (0 \leqq x \leqq s_1).$$

From our assumption $v(\xi) \leqq \eta_1$ and from the monotonicity of v we obtain

$$0 < v(x) < \beta\eta_1 = \delta \qquad (0 \leqq x \leqq s_1).$$

Taking the p occuring at the beginning of this proof to be equal to s_1, we find

$$\left|\frac{u'(x)}{u(x)}\right| \leqq 2M \qquad (0 \leqq x \leqq s_1).$$

Finally applying (3.7) to $x_0 = s_1$, $h_2 = 1 + f_2(\eta_1)$ we get $s_1 + [2(1 + f_2(\eta_1))^{\frac{1}{2}} + 4M]^{-1} \leqq \xi$ and this contradicts the definition of s_1. Indeed, by this definition and the definition of μ it holds

$$\xi = s_1 + 2\mu < s_1 + [2(1 + f_2(\eta_1))^{\frac{1}{2}} + 4M]^{-1}.$$

4. Conclusion

Theorem 1. Assume $\xi_0 > 0$ and assume that u, $v \in C^\circ([0, \xi_0]) \cap C^2((0, \xi_0])$ satisfy (1.1) and (1.2) for $0 < x \leqq \xi_0$ with $u(x) > 0$ $(0 \leqq x < \xi_0)$, $u(\xi_0) = 0$, $u'(0) = v'(0) = 0$, $v(0) > 0$. Then we have (with $\eta_0 = v(\xi_0)$)

$$\gamma[1 + f_2(\eta_0)]^{-\frac{1}{2}} < \xi_0 < \gamma.$$

Proof. From (1.4) and from the monotonicity of functions v, f_1, f_2 we obtain

$$1 < 1 + f_1(v(0)) \leqq f(x, u(x), v(x), u'(x), v'(x)) \leqq 1 + f_2(\eta_0) \qquad (0 \leqq x \leqq \xi_0)$$

It follows from Lemma 2 (cf. the proof of Lemma 3) that

$$\gamma[1 + f_2(\eta_0)]^{-\frac{1}{2}} \leqq \xi_0 \leqq \gamma[1 + f_1(v(0))]^{-\frac{1}{2}}.$$

Then we have

$$\gamma[1 + f_2(\eta_0)]^{-\frac{1}{2}} \leqq \xi_0 < \gamma.$$

Using the Sturmś method we prove that the equality cannot hold.

Put $u_0(x) = x^{-a}J_a(\gamma\xi_0^{-1}x)$, then

$$u_0'' + ax^{-1}u_0' + \gamma^2\xi_0^{-2}u_0 = 0$$

and

$$\int_0^{\xi_0} x^a[f(x, u(x), v(x), u'(x), v'(x)) - \gamma^2\xi_0^{-2}]\, u_0(x)\, u(x)\, dx = \int_0^{\xi_0} [x^a(u_0'(x)\, u(x) - - u_0(x)\, u'(x))]' dx = 0.$$

Now $1 + f_2(\eta_0) \leqq \gamma^2\xi_0^{-2}$ is impossible since $x^a > 0$, $u(x) > 0$, $u_0(x) > 0$ and $f(x, u(x), v(x), u'(x), v'(x)) - \gamma^2\xi_0^{-2} \leqq 1 + f_2(v(x)) - \gamma^2\xi_0^{-2} < 1 + f_2(\eta_0) - \gamma^2\xi_0^{-2}$ for $0 < x < \xi_0$. Therefore

$$1 + f_2(\eta_0) > \gamma^2\xi_0^{-2}.$$

Theorem 2. Assume $\xi_0 > 0$, $\eta_0 > 0$, $\gamma[1 + f_1(\eta_0)]^{-\frac{1}{2}} < \xi_0 < \gamma$. Then there exist functions $u, v \in C^0([0, \xi_0]) \cap C^2((0, \xi_0])$ that satisfy (1.1) and (1.2) for $0 < x \leqq \xi_0$, with

$$u(x) > 0 \ (0 \leqq x < \xi_0),\ u(\xi_0) = 0,\ u'(0) = v'(0) = 0,\ v(0) > 0,\ v(\xi_0) = \eta_0.$$

Proof. We choose numbers ξ_1, η_1 with

$$\xi_0 < \xi_1 < \gamma,\ \eta_0 < \eta_1,$$

and we take P according to Lemma 3.
We choose another number σ with $\sigma < P$,

$$0 < \sigma < G_2\{2\gamma^2\eta_0^{-1}[\eta_0 - F_1(\gamma^2\xi_0^{-2} - 1)]\} \tag{4.1}$$

(note that F_i and G_i denotes the inverse function of f_i and g_i, respectively (i = 1, 2)).

With the values of σ, P and η_1 we consider the rectangle in Fig. 1 and its image under the mapping Φ (see Sec. 2). If (p, q) lies on the lower edge E_1, the solution for v is identically zero, whence

$u(x) = p\Gamma(a + 1)\, 2^a x^{-a} J_a(x)$ $(0 \leqq x \leqq \gamma)$, and $\Phi(p, q)$ stays at the point $(\gamma, 0)$.

Next take (p, q) on the edge E_2, where $p = \sigma$, $0 < q \leqq \eta_1$. Put $\Phi(p, q) = (\xi, \eta)$. We have $0 \leqq u(x) \leqq \sigma$ $(0 \leqq x \leqq \xi)$ and v is positive and increasing (cf. Sec. 2). Therefore

$$v''(x) \leqq v''(x) + bx^{-1}v'(x) = g(x, u(x), v(x), u'(x), v'(x))\, v(x) \leqq$$
$$\leqq v(x)\, g_2(u(x)) \leqq \eta g_2(\sigma)$$

and by integration

$$\eta = v(\xi) = v(0) + v'(0)\,\xi + \int_0^\xi (\xi - t)\,v''(t)\,dt \leqq$$

$$\leqq v(0) + \eta g_2(\sigma)\frac{\xi^2}{2} \leqq v(0) + \frac{1}{2}\,\eta g_2(\sigma)\,\gamma^2,$$

whence

$$\eta\left(1 - \frac{1}{2}\,g_2(\sigma)\,\gamma^2\right) \leqq v(0). \tag{4.2}$$

Obviously it holds

$$1 < 1 + f_1(v(0)) \leqq f(x, u(x), v(x), u'(x), v'(x)) \leqq 1 + f_2(\eta) \qquad (0 \leqq x \leqq \xi)$$

and then from Lemma 2 we get

$$\xi \leqq \gamma[1 + f_1(v(0))]^{-\frac{1}{2}}$$

or respectively

$$0 < v(0) \leqq F_1(\gamma^2\xi^{-2} - 1), \tag{4.3}$$

From (4.2) and from (4.3) (note that $1 - \frac{1}{2}\,g_2(\sigma)\,\gamma^2 > 0$) we get

$$\eta \leqq \left[1 - \frac{1}{2}\,g_2(\sigma)\,\gamma^2\right]^{-1} F_1(\gamma^2\xi^{-2} - 1).$$

On the other hand we have, by (4.1):

$$\eta_0 > \left[1 - \frac{1}{2}\,g_2(\sigma)\,\gamma^2\right]^{-1} F_1(\gamma^2\xi_0^{-2} - 1).$$

So the image of the edge E_2 does not get to the right of the continuous curve representing graphically the continuous function

$$\eta = \left[1 - \frac{1}{2}\,g_2(\sigma)\,\gamma^2\right]^{-1} F_1(\gamma^2\xi^{-2} - 1),\ \xi \in (0, \gamma]$$

with the property $\eta \to 0$ for $\xi \to \gamma^-$, $\eta \to \infty$ for $\xi \to 0^+$, whereas the point (ξ_0, η_0) lies to the right of it.

The image of the edge E_3 lies entirely above the level $\eta = \eta_1$, simply because $q = \eta_1$ implies $v(\xi) > v(0) = \eta_1$. Finally, the image of E_4 is a curve that runs from some point above the level $\eta = \eta_1$ to the point $(\gamma, 0)$ without entering into rectangle

$$0 \leqq \xi \leqq \xi_1, \qquad 0 \leqq \eta \leqq \eta_1.$$

This we proved in Lemma 3 and depicted in Fig. 2. From the survey we offered it follows that the image of our rectangular contour has non-zero winding number with respect to the point (ξ_0, η_0). It follows that at least one interior point of the rectangle is mapped onto (ξ_0, η_0). This completes the proof.

From Theorem 1, Theorem 2 and from the definitions of functions f_1, f_2 it evidently follows that following assertions are true.

Theorem 3. For a solution of the boundary value problem **(I)**, **(II)** to exist it is necessary that

$$A < \left(\frac{\gamma}{t_0}\right)^2 < H_2^{(1)}(\eta_0)$$

and it is sufficient that

$$A < \left(\frac{\gamma}{t_0}\right)^2 < H_1^{(1)}(\eta_0),$$

where $A = H_1^{(1)}(0) = H_2^{(1)}(0)$.

Corollary 1. Suppose that function H_1 instead of assumptions $(\mathbf{A}_1)$, $(\mathbf{A}_2)$, $(\mathbf{A}_3)$ satisfies the conditions:

$$H_1(t, z_1, z_2, z_3, z_4) = H_0(z_2) \text{ for each } (t, z_1, z_2, z_3, z_4) \in [0, \tau] \times R_+^2 \times R^2$$

where the function H_0: $R_+ \to R_+$ with $H_0(0) \leqq \gamma^2 \tau^{-2} > 0$ satisfies one of the following conditions:

1. H_0 is a continuous increasing function on the interval R_+ and it is unbounded from above on this interval. For each compact set $[a_1, b_1] \times [a_2, b_2] \subset R_+^2$ there exists a constant $L > 0$ such that

$$|z_1 H_0(z_2) - \bar{z}_1 H_0(\bar{z}_2)| \leqq L[|z_1 - \bar{z}_1| + |z_2 - \bar{z}_2|]$$
$$((z_1, z_2)(\bar{z}_1, \bar{z}_2) \in [a_1, b_1] \times [a_2, b_2])$$

2. The function H_0 has a derivative $\frac{dH_0}{dz_2}$ on the interval R_+ satisfying inequalities $0 < c \leqq \frac{dH_0}{dz_2} \leqq C$ (where c, C are some constants) on this interval.
3. The function H_0 has a continuous derivative $\frac{dH_0}{dz_2}$ on the interval R_+ satisfying the inequality $0 < c \leqq \frac{dH_0}{dz_2}$ (where c is a constant) on this interval.

Then the problem **(I)**, **(II)** has a solution if and only if

$$H_0(0) < \left(\frac{\gamma}{t_0}\right)^2 < H_0(\eta_0).$$

Remark 1. In the paper [1] it is stated that the problem (**I**), (**II**) with $H_1(t, z_1, z_2, z_3, z_4) = A + Bz_2$ (where A, B are positive constants),

$$H_2(t, z_1, z_2, z_3, z_4) = z_1, \; a = b = t_0 = \eta_0 = 1$$

has a solution if and only if $A < \gamma^2(0) < A + B$, where $\gamma(0) \doteq 2{,}405$ is the first positive zero of the Bessel function $J_0(x)$. It is clear that this result is an easy consequence of Corollary 1.

By Theorem 3 the solvability of the problem (**I**), (**II**) depends on the value γ, where $\gamma = \gamma(\alpha)$ is the first positive zero of the Bessel function $J_\alpha(x)$ with $\alpha = \frac{1}{2}(a - 1)$. Then the solvability of this problem depends on the value $a \in R_+$. If we assume what has been stated above that $b \geqq 0$, $\tau \geqq t_0 > 0$ are given constants and H_1, H_2 are given functions satisfying the assumptions ($\mathbf{A}_1$), ($\mathbf{A}_2$), ($\mathbf{A}_3$) then the following problem can be solved: Is there at least one real number $a \geqq 0$ such that the problem (**I**), (**II**) has a solution?

Since $\gamma = \gamma(\alpha)$ regarded as a function of $\alpha \in \left[-\frac{1}{2}, \infty\right)$ is continuous, increasing and unbounded from above on the interval $\left[-\frac{1}{2}, \infty\right)$ (cf. [5, p. 125]), $\gamma\left(-\frac{1}{2}\right) = \frac{\pi}{2}$, so by using Theorem 3 we easily prove

Corollary 2. 1. Let there exist at least one function $H_1^{(1)}$ with the properties mentioned in the assumption ($\mathbf{A}_3$) and such that

$$H_1^{(1)}(\eta_0) > \left(\frac{\pi}{2t_0}\right)^2.$$

Then there exists a bounded interval $I \subset R_+$ such that the problem (**I**), (**II**) with a coefficient $a \in I$ has a solution.

2. Let there exist at least one function $H_2^{(1)}$ with properties mentioned in the assumption ($\mathbf{A}_3$) and such that

$$H_2^{(1)}(\eta_0) \leqq \left(\frac{\pi}{2t_0}\right)^2.$$

Then the problem (**I**), (**II**) has no solution for each $a \in R_+$.

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РЕЗЮМЕ

ОБ ОДНОЙ НЕЛИНЕЙНОЙ ДВУХТОЧЕЧНОЙ КРАЕВОЙ ЗАДАЧЕ

Йозеф Фульиер, Нитра

В статье обобщается одна краевая задача предложена голландским физиком. М. Й. Ц. ван Гемертом (в работе о газовом разряде), которую исследовал Н. Г. де Бруийн. Топологическим методом здесь доказывается достаточное условие и методом Штурма здесь также доказывается необходимое условие для разрешимости этой обобщенной задачи. В ней показано, что для довольно широкого класса краевых задач рассматриваемого типа совпадает необходимое условие для разрешимости этой краевой задачи с достаточным условием.

SÚHRN

O JEDNEJ NELINEÁRNEJ DVOJBODOVEJ OKRAJOVEJ ÚLOHE

Jozef Fulier, Nitra

V článku sa zovšeobecňuje špeciálna okrajová úloha sformulovaná holandským fyzikom M. J. C. van Gemertom (v práci o výboji v plyne), ktorú vyšetroval N. G. de Bruijn. Topologickou metódou je tu dokázaná postačujúca podmienka k tomu, aby existovalo riešenie tejto zovšeobecnenej okrajovej úlohy. Je tu tiež ukázané, že pre dosť širokú triedu okrajových úloh skúmaného typu je nevyhnutná podmienka pre existenciu riešenia danej okrajovej úlohy totožná s podmienkou postačujúcou.