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A REMARK ON THE METRIC STRUCTURE OF THE SPACE OF INTEGRABLY BOUNDED FUZZY VARIABLES

J. BÁN, Bratislava

1. Introduction and preliminaries

The study of the theory of fuzzy sets initiated by L. A. Zadeh has received considerable attention in recent years. An important role in this theory is played by the concept of fuzzy random variables introduced by Puri and Ralescu in [11]. This concept generalizes the concept of random sets and was defined as a tool for representing relationships between the outcomes of a random experiment and inexact data.

Puri and Ralescu [11] defined also the metric structure in the space of fuzzy subsets of R^n. The metric in the space of all integrably bounded random sets was introduced by Hiai and Umegaki [8].

Using these results we define the metric in the space of integrably bounded fuzzy random variables and we show that this metric space is complete and it can be embedded isometrically into a normed space (which is a generalization of the results due to Rådström [13] and Puri and Ralescu [10]).

Let $\mathscr{X}$ be a separable Banach space with the norm $\| . \|$ and let A, B be two nonempty bounded subsets of $\mathscr{X}$. The Hausdorff distance between A and B is defined by

$$\mathrm{h}(A, B) = \max\{\sup_{a \in A} \inf_{b \in B} \|a - b\|, \sup_{b \in B} \inf_{a \in A} \|a - b\|\}.$$

If $K(\mathscr{X})$ denotes the family of all nonempty compact subsets of $\mathscr{X}$, it is well known that $(K(\mathscr{X}), \mathrm{h})$ is a complete separable metric space. For $A \in K(\mathscr{X})$ denote by

$$|A| = \sup_{a \in A} \|a\| = \mathrm{h}(A, \{0\}).$$

By $\mathrm{co}K(\mathscr{X})$ we denote the family of all nonempty compact and convex subsets of $\mathscr{X}$.

The linear structure in $K(\mathscr{X})$ can be defined by the Minkowski operations of addition and scalar multiplication:

$A + B = \{a + b \colon a \in A,\ b \in B\}$ and $\lambda A = \{\lambda a \colon a \in A\}$

where A, $B \in K(\mathscr{X})$ and $\lambda \in \mathrm{R}$.

Let $(\Omega, \mathscr{A}, \mathrm{P})$ be a probability space with $\mathscr{A}$ a σ-field of measurable subsets of the set Ω and a probability measure P.

Let F: $\Omega \to K(\mathscr{X})$. The function F is called a (compact) random set if $\mathrm{F}^{-1}(B) \in \mathscr{A}$ for each Borel set B in metric space $(K(\mathscr{X}), \mathrm{h})$ (for the measurability of random sets see also [8]). Let $\mathrm{L}^1(\Omega, \mathscr{X})$ denote the Banach space of (equivalence classes of) measurable functions f: $\Omega \to \mathscr{X}$ such that the norm

$$\| \mathrm{f} \|_{\mathrm{L}^1} = \int_\Omega \| \mathrm{f} \| \, d\mathrm{P}$$

is finite. By $\mathrm{L}^1(\Omega, \mathrm{R})$ we denote the usual Banach space of real-valued measurable functions.

A measurable function f: $\Omega \to \mathscr{X}$ is called a (measurable) selection of F if $\mathrm{f}(\omega) \in \mathrm{F}(\omega)$ P-a.e. Denote by

$$\mathrm{S(F)} = \{\mathrm{f} \in \mathrm{L}^1(\Omega, \mathscr{X}) \colon \mathrm{f} \text{ is a selection of } \mathrm{F}\}$$

An expected value EF of a random set F was defined in [1] in the following way:

$$\mathrm{EF} = \{\mathrm{Ef} \colon \mathrm{f} \in \mathrm{S(F)}\}.$$

Another definition of the expectation can be found in [6]. These definitions were shown to be equivalent by Byrne [5]. A random set is called integrably bounded if there exists a nonnegative real-valued function $\xi \in \mathrm{L}^1(\Omega, \mathrm{R})$ such that $\| x \| \le \xi(\omega)$ for any x and ω with $x \in \mathrm{F}(\omega)$. Since the function $\omega \mapsto |\mathrm{F}(\omega)|$ is measurable, it is clear that F is integrably bounded if and only if

$$\int_\Omega |\mathrm{F}(\omega)| \, d\mathrm{P} < \infty.$$

Moreover, $\int_\Omega |\mathrm{F}(\omega)| \, d\mathrm{P} = \sup_{\mathrm{f} \in \mathrm{S(F)}} \int_\Omega \| \mathrm{f}(\omega) \| \, d\mathrm{P}$ if $\mathrm{S(F)} \neq \emptyset$ and thus F is integrably bounded if and only if S(F) is nonempty and bounded in $\mathrm{L}^1(\Omega, \mathscr{X})$ ([8], Theorem 3.2.)

The space of all integrably bounded random sets with compact values will be denoted by $\mathscr{L}(\Omega, \mathscr{X})$, where two functions F_1, $\mathrm{F}_2 \in \mathscr{L}(\Omega, \mathscr{X})$ are considered to be identical if $\mathrm{F}_1(\omega) = \mathrm{F}_2(\omega)$ P-a.e.

For F_1, $\mathrm{F}_2 \in \mathscr{L}(\Omega, \mathscr{X})$ it is easy to see that

$$\mathrm{h}(\mathrm{F}_1(\omega), \mathrm{F}_2(\omega)) \le |\mathrm{F}_1(\omega)| + |\mathrm{F}_2(\omega)|$$

and thus the function $\omega \mapsto h(F_1(\omega), F_2(\omega))$ is in $L^1(\Omega, R)$ and we can define

$$l(F_1, F_2) = \int_\Omega h(F_1(\omega), F_2)) \, dP.$$

In [8], Theorem 3.3. it was proved that $(\mathscr{L}(\Omega, \mathscr{X}), l)$ is a complete metric space.

2. Fuzzy random variables

If M is a set, a fuzzy subset of M is a function $u: M \to [0, 1]$. For any fuzzy subset $u: M \to [0, 1]$ denote

$$u^\alpha = \{m \in M: u(m) \geq \alpha\}, \qquad \alpha \in [0, 1].$$

Let $\mathscr{F}_0(\mathscr{X})$ denote the space of all fuzzy subsets $u: \mathscr{X} \to [0, 1]$ such that:

(1) u^α is a compact subset of $\mathscr{X}$ for every $\alpha > 0$

(2) $u^1 = \{x \in \mathscr{X}: u(x) = 1\}$ is nonempty
and let $\mathscr{F}_1(\mathscr{X}) = \{u \in \mathscr{F}_0(\mathscr{X}): u^\alpha \text{ is convex}, \alpha > 0\}$.

We use a concept of fuzzy random variables introduced by Puri and Ralescu [11].
A fuzzy random variable is a function $X: \Omega \to \mathscr{F}_0(\mathscr{X})$ such that for every $\alpha > 0$ the function $X^\alpha: \Omega \to K(\mathscr{X})$ defined by $X^\alpha(\omega) = \{x \in \mathscr{X}: X(\omega)(x) \geq \alpha\} = (X(\omega))^\alpha$ is a (measurable) random set. The functions X^α are called α-cuts of X.

Lemma 1: Let X and Y be two fuzzy random variables. Then $X(\omega) = Y(\omega)$ P-a.e. if and only if $X^\alpha(\omega) = Y^\alpha(\omega)$ P-a.e. for every $\alpha > 0$.

Proof: If $X(\omega) = Y(\omega)$ P-a.e. then clearly also $X^\alpha(\omega) = Y^\alpha(\omega)$ P-a.e., $\alpha > 0$. Conversely, let Q denote the set of all rational numbers in [0, 1].
Put $A_\alpha = \{\omega \in \Omega: X^\alpha(\omega) = Y^\alpha(\omega)\}$.

Since $X^\alpha(\omega) = Y^\alpha(\omega)$ P-a.e. then $P(A_\alpha) = 1$ and

$$P\left(\bigcap_{\alpha \in Q} A_\alpha\right) = 1 - P\left(\bigcup_{\alpha \in Q} A_\alpha^c\right) \geq 1 - \sum_{\alpha \in Q} P(A_\alpha^c) = 1.$$

Let $A = \bigcap_{\alpha \in Q} A_\alpha$.
For every $\omega \in A$ and $\alpha \in Q$ it holds $X^\alpha(\omega) = Y^\alpha(\omega)$ and thus

$$X(\omega)(x) \geq \alpha \text{ if and only if } Y(\omega)(x) \geq \alpha, \ x \in \mathscr{X}.$$

Let there exist $\omega \in A$ and $x \in \mathscr{X}$ such that e.g. $X(\omega)(x) > Y(\omega)(x)$. Then there exists $\alpha \in Q$ such that $X(\omega)(x) > \alpha > Y(\omega)(x)$ which means that $X(\omega)(x) \geq \alpha$ but it does not hold $Y(\omega)(x) \geq \alpha$ which is a contradiction. Q. E. D.

A fuzzy random variable is called integrably bounded if X^α is integrably bounded random set for all $\alpha \in (0, 1]$. The space of all integrably bounded fuzzy random variables we denote by $FV(\Omega, \mathscr{F}_0(\mathscr{X}))$.

The expected value EX of a fuzzy random variable X is defined as the unique fuzzy set which satisfies the property

$$(EX)^\alpha = E(X^\alpha)$$

for every $\alpha \in (0, 1]$. The proof of the existence of this integral for any integrably bounded fuzzy random variable is to be found in [11] (for the case $\mathscr{X} = R^n$, but for this general case the proof works in the same way). The proof in the question is based on the set representation of fuzzy sets which can be formulated as follows ([9]):

Lemma 2: Let M be a set and let $\{M_\alpha, \alpha \in [0, 1]\}$ be a family of subsets of M such that

(1) $M_0 = M$

(2) $\alpha \leq \beta$ implies $M_\alpha \supseteq M_\beta$

(3) $\alpha_1 \leq \alpha_2 \leq \ldots, \lim\limits_{n \to \infty} \alpha_n = \alpha$ implies $M_\alpha = \bigcap\limits_{n=1}^{\infty} M_{\alpha_n}$

Then the function $u: M \to [0, 1]$ defined by $u(x) = \sup\{\alpha: x \in M_\alpha\}$ has the property that $\{x \in M: u(x) \geq \alpha\} = M_\alpha$ for every $\alpha \in [0, 1]$.

A natural generalization of the Hausdorff distance to the space $\mathscr{F}_0(\mathscr{X})$ is the metric d introduced in [11]. For $u, v \in \mathscr{F}_0(\mathscr{X})$ define by

$$d(u, v) = \sup_{\alpha > 0} h(u^\alpha, v^\alpha).$$

In the same paper it was shown that $(\mathscr{F}_0(\mathscr{X}), d)$ is a complete metric space. The metric structure in $FV(\Omega, \mathscr{F}_0(\mathscr{X}))$ can be defined similarly.

For $X, Y \in FV(\Omega, \mathscr{F}_0(\mathscr{X}))$ define

$$D(X, Y) = \sup_{\alpha > 0} \int_\Omega h(X^\alpha(\omega), Y^\alpha(\omega))\, dP.$$

In the next two fuzzy random variables $X, Y \in FV(\Omega, \mathscr{F}_0(\mathscr{X}))$ are considered to be identical if $X(\omega) = Y(\omega)$ P-a.e.

Lemma 3: $(FV(\Omega, \mathscr{F}_0(\mathscr{X})), D)$ is a metric space.

Proof: $D(X, Y) = 0$ implies $\int_\Omega h(X^\alpha(\omega), Y^\alpha(\omega))\, dP = 0$ for every $\alpha > 0$ and thus $X^\alpha(\omega) = Y^\alpha(\omega)$ P-a.e. By Lemma 1 we have $X(\omega) = Y(\omega)$ P-a.e.
The properties $D(X, Y) = D(Y, X)$ and $D(X, Y) \leq D(X, Z) + D(Z, Y)$ are clear from the definition of the metric D.

Q. E. D.

Theorem 1: The metric space $(\mathrm{FV}(\Omega, \mathscr{F}_0(\mathscr{X})), \mathrm{D})$ is complete.

Proof: Let $\{X_n\}$ be a Cauchy sequence in $\mathrm{FV}(\Omega, \mathscr{F}_0(\mathscr{X}))$. Take any $\alpha \in (0, 1]$. The sequence $\{X_n^\alpha\}$, where X_n^α is an α-cut of fuzzy random variable X_n, is a Cauchy sequence in $\mathscr{L}(\Omega, \mathscr{X})$.

Since the space $(\mathscr{L}(\Omega, \mathscr{X}), l)$ is complete then there exists a random set $X^\alpha \in \mathscr{L}(\Omega, \mathscr{X})$ such that

$$X_n^\alpha \xrightarrow{l} X^\alpha$$

where the convergence in the metric l is uniform with respect to α.

For every $\omega \in \Omega$ consider the family $\{X^\alpha(\omega), \alpha \in [0, 1]\}$ where $X^0(\omega) = \mathscr{X}$.

The purpose of the next part of the proof is to show that the family $\{X^\alpha(\omega), \alpha \in [0, 1]\}$ satisfies the conditions (2) and (3) of Lemma 2. The proof of these properties is similar to the one used in [11].

Denote by $\varrho(A, B) = \sup_{a \in A} \inf_{b \in B} \| a - b \|$.

Note that ϱ is a semimetric and $A \subseteq \mathrm{cl}\, B$ if and only if $\varrho(A, B) = 0$ where $\mathrm{cl}\, B$ is the closure of the set B.

(2) Let $\alpha \leq \beta$. Then

$$\varrho(X^\beta(\omega), X^\alpha(\omega)) \leq \varrho(X^\beta(\omega), X_n^\beta(\omega)) + \varrho(X_n^\beta(\omega), X_n^\alpha(\omega)) + \varrho(X_n^\alpha(\omega), X^\alpha(\omega))$$

Since $X_n^\gamma \xrightarrow{l} X^\gamma$ then $X_n^\gamma(\omega) \xrightarrow{h} X^\gamma(\omega)$ P-a.e. for $\gamma = \beta, \alpha$, thus the first and the third term tend to zero for almost every $\omega \in \Omega$. The second term $\varrho(X_n^\beta(\omega), X_n^\alpha(\omega)) = 0$ because $X_n^\beta(\omega) \subseteq X_n^\alpha(\omega)$. Thus $X^\beta(\omega) \subseteq X^\alpha(\omega)$ P-a.e.

(3) Let $\alpha_n \nearrow \alpha$. By the property (2) it is clear that $X^\alpha(\omega) \subseteq \bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega)$ P-a.e. and thus it is enough to prove that

$$\varrho\left(\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega), X^\alpha(\omega)\right) = 0.$$

$$\text{Clearly } \varrho\left(\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega), X^\alpha(\omega)\right) \leq \varrho\left(\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega), \bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega)\right) +$$

$$+ \varrho\left(\bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega), X_j^\alpha(\omega)\right) + \varrho(X_j^\alpha(\omega), X^\alpha(\omega))$$

for any fixed j. Since $\bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega) = X_j^\alpha(\omega)$ then

$$\varrho\left(\bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega), X_j^\alpha(\omega)\right) = 0.$$

The third term converges to zero because $X_j^\alpha \xrightarrow{l} X^\alpha$. Let us show that also the first term is equal to zero. For every $m \geq 1$ it holds

$$\varrho\left(\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega), \bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega)\right) \leq \varrho\left(\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega), X^{\alpha_m}(\omega)\right) +$$

$$+ \varrho(X^{\alpha_m}(\omega), X_j^{\alpha_m}(\omega)) + \varrho\left(X_j^{\alpha_m}(\omega), \bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega)\right).$$

Since $\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega) \subseteq X^{\alpha_m}(\omega)$ then $\varrho\left(\bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega), X^{\alpha_m}(\omega)\right) = 0.$

Clearly $\varrho(X^{\alpha_m}(\omega), X_j^{\alpha_m}(\omega)) < \varepsilon$ for $j \geq j_0$.

Note that the index j_0 does not depend on m (since $X_n^\alpha \xrightarrow{h} X^\alpha$ uniformly with respect to α). Finally, by the argument similar to the one above, we have

$$\varrho\left(X_j^{\alpha_m}(\omega), \bigcap_{n=1}^{\infty} X_j^{\alpha_n}(\omega)\right) < \varepsilon \text{ for } m \geq m_0.$$

Thus $X^\alpha(\omega) \supseteq \bigcap_{n=1}^{\infty} X^{\alpha_n}(\omega)$ P-a.e. and there exists $E \in \mathscr{A}$ such that $P(E) = 0$ and for every $\omega \in \Omega \setminus E$ the family $\{X^\alpha(\omega), \alpha \in [0, 1]\}$ satisfies the conditions of **Lemma 2** and we may apply it. Put

$$X(\omega) = u(x) \in \mathscr{F}_0(\mathscr{X}) \text{ where } u(x) = \sup\{\alpha: x \in X^\alpha(\omega)\} \quad \text{if } \omega \in \Omega \setminus E$$

$$= I_{\{0\}} \quad \text{if } \omega \in E,$$

where $I_{\{0\}}$ is the indicator function of the set $\{0\}$.

Let us show that $X_n \xrightarrow{D} X$.

Since $\{X_n\}$ is a Cauchy sequence then for every $\varepsilon > 0$ and $n, m \geq n_0$ it holds $D(X_n, X_m) < \varepsilon$.

Let n $(n \geq n_0)$ be fixed. Then, using the Lebesgue convergence theorem for random variables, we obtain

$$l(X_n^\alpha, \lim_{m\to\infty} X_m^\alpha) = \int_\Omega h(X_n^\alpha(\omega), \lim_{m\to\infty} X_m^\alpha(\omega))\, dP =$$

$$= \int_\Omega \lim_{m\to\infty} h(X_n^\alpha(\omega), X_m^\alpha(\omega))\, dP = \lim_{m\to\infty} \int_\Omega h(X_n^\alpha(\omega), X_m^\alpha(\omega))\, dP = \lim_{m\to\infty} l(X_n^\alpha, X_m^\alpha)$$

and

$$l(X_n^\alpha, X^\alpha) = \lim_{m\to\infty} (X_n^\alpha, X_m^\alpha) \le \lim_{m\to\infty} \sup_{\alpha>0} l(X_n^\alpha, X_m^\alpha) = \lim_{m\to\infty} D(X_n, X^m) < \varepsilon$$

for every $\alpha > 0$.

Thus $D(X_n, X) = \sup_{\alpha>0} l(X_n^\alpha, X^\alpha) \le \varepsilon$ for $n \ge n_0$. Q. E. D.

Using Lemma 2, the linear structure in $\mathscr{F}_0(\mathscr{X})$ can be defined as follows:

$$(u + v)(x) = \sup\{\alpha: x \in u^\alpha + v^\alpha\}$$

$$\begin{aligned}(\lambda u)(x) &= \sup\{\alpha: x \in \lambda u^\alpha\} = u(\lambda^{-1}. x) && \text{if } \lambda \neq 0\\ &= 0 && \text{if } \lambda = 0 \text{ and } x \neq 0\\ &= \sup_{y\in\mathscr{X}} u(y) && \text{if } \lambda = 0 \text{ and } x = 0\end{aligned}$$

for u, $v \in \mathscr{F}_0(\mathscr{X})$, $\lambda \in R$.

In [10] it was shown that the metric space $(\mathscr{F}_1(\mathscr{X}), d)$ can be embedded isometrically into a Banach space ([10], Theorem 2.2.). To extend this result to the space $(FV(\Omega, \mathscr{F}_1(\mathscr{X})), D)$ we need to define the linear structure in $FV(\Omega, \mathscr{F}_1(\mathscr{X}))$:

$$(X + Y)(\omega) = X(\omega) + Y(\omega) \text{ and } (\lambda . X)(\omega) = \lambda . X(\omega)$$

for X, $Y \in FV(\Omega, \mathscr{F}_1(\mathscr{X}))$, $\lambda \in R$.

It is very easy to see that for any X, Y, $Z \in FV(\Omega, \mathscr{F}_1(\mathscr{X}))$ it holds

(a) $X + Y = Z + Y$ implies $X = Z$

(b) $D(X + Z, Y + Z) = D(X, Y)$

(see also [10], Proposition 2.2.) and thus the embedding theorem for $FV(\Omega, \mathscr{F}_1(\mathscr{X}))$ can be stated in the same way as [10], Theorem 2.2.

Theorem 2: There exists a normed space $\mathscr{N}$ such that $(FV(\Omega, \mathscr{F}_1(\mathscr{X})), D)$ can be embedded isometrically into $\mathscr{N}$ in such a way that:

a) addition in $L(\Omega, \mathscr{N})$ induces addition in $FV(\Omega, \mathscr{F}_1(\mathscr{X}))$

b) multiplication by nonnegative real-valued integrable function ξ in $L(\Omega, \mathscr{N})$ induces the corresponding operation in $FV(\Omega, \mathscr{F}_1(\mathscr{X}))$

For the construction of the space $\mathscr{N}$ and an embedding see [10], pages 555—556.

Note that some martingale convergence theorems in metric D were proved in [4] and the ergodic theorem was proved in [3].

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Author's address:

Ján Bán
Katedra teórie pravdepodobnosti
a matematickej štatistiky MFF UK
Mlynská dolina
842 15 Bratislava

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РЕЗЮМЕ

ЗАМЕЧАНИЕ К МЕТРИЧЕСКОЙ СТРУКТУРЕ ПРОСТРАНСТВА ИНТЕГРАЛЬНО-ОРГАНИЧЕННЫХ СЛУЧАЙНЫХ ВЕЛИЧИН

Ян Бан, Братислава

В пространстве интегрально-ограниченных случайных величин построена метрика, относительно которой, как показано в работе, рассмотриваемое пространство полно. Более того, в статье доказано, что данное метрическое пространство можно изометрически погружить в Банахово пространство сохраняя линейную заданово пространства.

SÚHRN

POZNÁMKA K METRICKEJ ŠTRUKTÚRE PRIESTORU INTEGROVATEĽNE OHRANIČENÝCH FUZZY NÁHODNÝCH PREMENNÝCH

Ján Bán, Bratislava

V priestore integrovateľne ohraničených fuzzy náhodných premenných je definovaná metrika, vzhľadom na ktorú je tento priestor úplný. Navyše je dokázané, že takýto metrický priestor sa dá izometricky vnoriť do nejakého Banachovho priestoru tak, že sa zachováva lineárna štruktúra obidvoch priestorov.