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NECESSARY AND SUFFICIENT CONDITIONS FOR OSCILLATIONS OF FIRST ORDER DELAY DIFFERENTIAL EQUATIONS AND INEQUALITIES

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1 Introduction

The purpose of this paper is to examine the oscillatory nature of linear delay differential equations of the form

$$x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) = 0, \quad t \geqq a > 0 \tag{1}$$

and

$$x'(t) + (t \ln t)^{-1} \sum_{i=1}^{n} p_i x(t^{\sigma_i}) = 0, \quad t \geqq a > 1, \tag{2}$$

where $p_i > 0$, $0 < \sigma_i \leqq 1$, $i = 1, 2, \ldots, n$, are constants. More precisely, we shall derive necessary and sufficient conditions under which all solutions of the retarded differential equations (1) and (2) oscillate. In proving our results we shall use the method developed by Ladas, Sficas and Stavroulakis in [8].

Recently, some authors (Tramov [12], Ladas, Sficas and Stavroulakis [8], Arino, Győri and Jawhari [1], Hunt and Yorke [4]) have independently derived the necessary and sufficient conditions for the oscillation of all solutions of the delay differential equation

$$x'(t) + \sum_{i=1}^{n} p_i x(t - \tau_i) = 0, \quad t \geqq a, \tag{3}$$

where p_i and τ_i are positive constants. According to their results all solutions of Eq. (3) oscillate if and only if the corresponding characteristic equation

$$-\lambda + \sum_{i=1}^{n} p_i e^{\lambda \tau_i} = 0 \tag{4}$$

has no real roots. Since the direct investigation of Eq. (4) may be a difficult problem itself, the authors mentioned above have developed also some explicit sufficient conditions which ensure that Eq. (4) has no real roots, so that all the solutions of Eq. (3) are oscillatory.

Surprisingly, however, there are not many results in the literature on the oscillation of delay differential equations of the form (1) and (2) which would be sharp enough to provide the characterization of the oscillation of all solutions. To the best of the author's knowledge, Čanturia [2] and Nadareišvili [10] are the only references related to the subject of this paper.

In what follows, the oscillatory character of the solution $x(t)$ of (1) or (2) defined on an interval $[t_x, \infty)$, $t_x \geqq a$, is considered in the usual sense, that is, $x(t)$ is said to be oscillatory if it has arbitrarily large zeros in $[t_x, \infty)$ and it is said to be nonoscillatory otherwise.

As it is customary, we shall say that a continuous real-valued function $u(t)$ defined on an interval $[t_u, \infty)$ eventually has some property if there is a $T \geqq t_u$ such that $u(t)$ has this property on $[T, \infty)$.

2 Main results

The problem of the characterization of oscillations of Eqs. (1) and (2) can be examined in a variety of ways. Our approach here patterns after that in [8] and requires only elementary tools.

We begin with a simple lemma which will be needed in the proof of our main theorem.

Lemma. Let $x(t)$ and $y(t)$ be the nonoscillatory solutions of the retarded differential inequalities

$$\operatorname{sgn} x(\sigma t)\{x'(t) + pt^{-1}x(\sigma t)\} \leqq 0, \quad t \geqq a > 0, \tag{5}$$

and

$$\operatorname{sgn} y(t^{\sigma})\{y'(t) + p(t \ln t)^{-1}y(t^{\sigma})\} \leqq 0, \quad t \geqq a > 1, \tag{6}$$

where $p > 0$, $0 < \sigma < 1$, defined on the intervals $[\sigma a, \infty)$ and $[a^{\sigma}, \infty)$, respectively. Then

$$|x(t)| \geqq \left(\frac{p \ln 1/\sigma}{2}\right)^2 |x(\sigma t)|, \quad t \geqq \sigma^{-3/2}a, \tag{7}$$

and

$$|y(t)| \geqq \left(\frac{p \ln 1/\sigma}{2}\right)^2 |y(t^{\sigma})|, \quad t \geqq a^{\sigma^{-3/2}}. \tag{8}$$

Proof. We prove only the part concerning the inequality (5) (the proof for (6) is similar).

Without loss of generality we may assume that the solution $x(t)$ of (5) is positive on $[\sigma a, \infty)$. Then the inequality (5) becomes

$$x'(t) + pt^{-1}x(\sigma t) \leqq 0, \quad t \geqq a. \tag{9}$$

Let $s \geqq \sigma^{-1}a$ be given. Integrating both sides of (9) from s to $\sigma^{-1/2}s$ and using the fact that $x(t)$ is decreasing on $[a, \infty)$, we obtain that

$$-x(s) + \frac{p \ln 1/\sigma}{2} x(\sigma^{1/2}s) \leqq$$

$$\leqq x(\sigma^{-1/2}s) - x(s) + \frac{p \ln 1/\sigma}{2} x(\sigma^{1/2}s) \leqq 0 \tag{10}$$

for $s \geqq \sigma^{-1}a$. For given $t \geqq \sigma^{-3/2}a$ we apply (10) to $s = \sigma^{1/2}t$ and to $s = t$ and get

$$x(\sigma^{1/2}t) \geqq [(p \ln 1/\sigma)/2]x(\sigma t)$$

and

$$x(t) \geqq [(p \ln 1/\sigma)/2]x(\sigma^{1/2}t)$$

for $t \geqq \sigma^{-3/2}a$. Combining these inequalities we get the desired relation (7).

Theorem 1. A necessary and sufficient condition for all the solutions of Eq. (1) (or Eq. (2)) to be oscillatory is that

$$-\alpha + \sum_{i=1}^{n} p_i \sigma_i^{-\alpha} > 0 \tag{11}$$

for all $\alpha > 0$.

Proof. We consider only Eq. (1).

(The "only if" part.) Assume to the contrary that (11) does not hold. Then there exists an $\alpha_0 > 0$ such that

$$-\alpha_0 + \sum_{i=1}^{n} p_i \sigma_i^{-\alpha_0} = 0$$

and so Eq. (1) has a nonoscillatory solution $x(t) = t^{-\alpha_0}$.

(The "if" part.) On the other hand, let (11) hold and assume for the sake of contradiction that there exists an eventually positive solution $x(t)$ of Eq. (1).

When all σ_i, $i = 1, 2, \ldots, n$, are equal to 1, then (11) obviously does not hold. Thus, we may assume without any loss of generality that $\sigma_n = \min\{\sigma_1, \ldots, \sigma_n\} < 1$.

Define the set

$$A = \{\alpha > 0\colon x'(t) + \alpha t^{-1}x(t) < 0 \quad \text{eventually}\}.$$

From (1) we have

$$x'(t) + p_n t^{-1} x(t) < x'(t) + p_n t^{-1} x(\sigma_n t) \leqq 0$$

eventually, so that $p_n \in A$. Consequently, the set A is non-empty.

Taking into account that $x(t)$ is decreasing and using Lemma, we find

$$0 = x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) \leqq x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_n t) \leqq$$

$$\leqq x'(t) + (2/(p_n \ln 1/\sigma_n))^2 t^{-1} \sum_{i=1}^{n} p_i x(t)$$

eventually, which proves that $(2/(p_n \ln 1/\sigma_n))^2 \sum_{i=1}^{n} p_i$ is an upper bound of A. Since A is non-empty and bounded from above, it has the supremum. Denote this supremum by $\bar{\alpha}$.

Let $\alpha \in A$ be given and consider the function $u(t) = t^{\alpha} x(t)$. Then

$$u'(t) = t^{\alpha}(x'(t) + (\alpha/t)x(t)) < 0$$

for all large t, which implies that u is eventually decreasing. From (1) we obtain

$$0 = x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) = x'(t) + t^{-1} \sum_{i=1}^{n} p_i (\sigma_i t)^{-\alpha} u(\sigma_i t) >$$

$$> x'(t) + t^{-1} \sum_{i=1}^{n} p_i \sigma_i^{-\alpha} t^{-\alpha} u(t) = x'(t) + t^{-1} \sum_{i=1}^{n} p_i \sigma_i^{-\alpha} x(t)$$

for all large t and, consequently, $\sum_{i=1}^{n} p_i \sigma_i^{-\alpha} \in A$.

This implies that $\sum_{i=1}^{n} p_i \sigma_i^{-\alpha} \leqq \bar{\alpha}$ and since $\alpha \in A$ was arbitrary, we conclude that

$$\sum_{i=1}^{n} p_i \sigma_i^{-\bar{\alpha}} \leqq \bar{\alpha}.$$

But this is the contradiction to (11).

The proof in the case when $x(t)$ is an eventually negative solution of Eq. (1) (or (2)) is similar and we omit it.

Following the idea of Arino, Györi and Jawhari in [1], Theorem 1 can be restated also as follows.

Theorem 2. All the solutions of Eq. (1) (or (2)) oscillate if and only if

$$-\alpha_0 + \sum_{i=1}^{n} p_i \sigma_i^{-\alpha_0} > 0, \tag{12}$$

where α_0 is a unique solution of

$$\sum_{i=1}^{n} p_i (\ln 1/\sigma_i)\sigma_i^{-\alpha} = 1. \tag{13}$$

Proof. Denote

$$F(\alpha) = -\alpha + \sum_{i=1}^{n} p_i \sigma_i^{-\alpha}, \quad \alpha \in R.$$

Since

$$F''(\alpha) = \sum_{i=1}^{n} p_i (\ln 1/\sigma_i)^2 \sigma_i^{-\alpha} > 0, \quad \alpha \in R,$$

$F(\alpha)$ is a convex function and it suffices to show that the minimum of $F(\alpha)$ is a positive value. Looking for this minimum we get the equation

$$F'(\alpha) = -1 + \sum_{i=1}^{n} p_i (\ln 1/\sigma_i)\sigma_i^{-\alpha} = 0$$

which is nothing else than (13).

On the other hand, from the first part of the proof of Theorem 1 it follows that if all the solutions of Eq. (1) are oscillatory, then (11) holds in fact for all real α. Thus, in particular, (12) is true for the real number α_0 given as a unique solution of (13). The proof is complete.

Due to their transcendental nature, neither (11) nor (13) are easily tractable. Therefore, we proceed further and derive a set of explicit sufficient conditions for oscillation of Eqs. (1) and (2) in terms of the coefficients p_i and σ_i only. The advantage of working with these explicit conditions rather than (11) and (13) is obvious.

Theorem 3. Each of the following conditions implies that every solution of Eq. (1) (or (2)) is oscillatory:

(a) $p_i \ln 1/\sigma_i > 1/e$ for some $i \in \{1, 2, \ldots, n\}$;

(b) $\sum_{i=1}^{n} p_i (\ln 1/\sigma) > 1/e$, where $\sigma = \max\{\sigma_i, i = 1, \ldots, n\}$;

(c) $n\Delta_s(p_1 \ln 1/\sigma_1, \ldots, p_n \ln 1/\sigma_n) > 1/e$ for some $s < 1$, where $\Delta_s(a_1, \ldots, a_n)$, $a_i \geqq 0$, $i = 1, \ldots, n$, is defined by $\Delta_s(a_1, \ldots, a_n) = \left[(1/n) \sum_{i=1}^{n} a_i^s\right]^{1/s}$ for $s \neq 0$ and $\Delta_0(a_1, \ldots, a_n) = \lim_{s \to 0} \Delta_s(a_1, \ldots, a_n)$;

(d) $\sum_{i=1}^{n} p_i \ln 1/\sigma_i > 1/e$;

(e) $\left(\prod_{i=1}^{n} p_i\right)^{1/n} \left(\sum_{i=1}^{n} \ln 1/\sigma_i\right) > 1/e$.

Proof. It is easy to see that each of the conditions (a) and (b) implies (d). Moreover, since $\sum_{i=1}^{n} p_i \ln 1/\sigma_i = n\Delta_1(p_1 \ln 1/\sigma_1, \ldots, p_n \ln 1/\sigma_n)$ and the function Δ_s is nondecreasing in s for every fixed $2n$-parameter family (p_i, σ_i), $i = 1, \ldots, n$, $p_i > 0$, $0 < \sigma_i \leqq 1$, (see [3, p. 175]), the condition (c) also implies (d). Thus, it suffices to prove (d) and (e). In both cases we shall show that the condition (11) of Theorem 1 is satisfied.

Proof of (d): Let us consider the function

$$G(\alpha) = -1 + \sum_{i=1}^{n} p_i \sigma_i^{-\alpha} \alpha^{-1}, \quad \alpha > 0.$$

Denote by J the set of all i, $1 \leqq i \leqq n$, such that $\sigma_i \neq 1$. Minimizing every $p_i \sigma_i^{-\alpha} \alpha^{-1}$, $i \in \mathrm{J}$, by setting $\alpha = (\ln 1/\sigma_i)^{-1}$ and omitting in $G(\alpha)$ the members with $\sigma_i = 1$, we find

$$G(\alpha) \geqq -1 + \sum_{i \in \mathrm{J}} p_i \ln(1/\sigma_i)e = -1 + \sum_{i=1}^{n} p_i \ln(1/\sigma_i)e > 0$$

for $\alpha > 0$. Consequently,

$$\Gamma(\alpha) > 0 \quad \text{for all } \alpha > 0,$$

so that the condition (11) is satisfied.

Proof of (e): Let $G(\alpha)$ be defined as in the proof of (d). From the arithmetic mean-geometric mean inequality it follows that

$$G(\alpha) \geqq -1 + n\left(\prod_{i=1}^{n} p_i \sigma_i^{-\alpha} \alpha^{-1}\right)^{1/n} =$$

$$= -1 + \frac{n}{\alpha}\left(\prod_{i=1}^{n} p_i\right)^{1/n} \exp\left(\frac{\alpha}{n} \sum_{i=1}^{n} \ln 1/\sigma_i\right)$$

for $\alpha > 0$. Differentiating the last expression with respect to α we find that it is minimized at $\alpha = n\left(\sum_{i=1}^{n} \ln 1/\sigma_i\right)^{-1}$. Thus,

$$G(\alpha) \geqq -1 + \left(\sum_{i=1}^{n} \ln 1/\sigma_i\right)\left(\prod_{i=1}^{n} p_i\right)^{1/n} e > 0$$

for every $\alpha > 0$ and the proof is complete.

Remark 1. The conditions (d) and (e) are independent. We illustrate this fact by the following examples.

Consider the equation

$$x'(t) + \frac{1}{9t}x(e^{-1}t) + \frac{1}{t}x(e^{-1/9}t) = 0, \quad t \geqq a > 0. \tag{14}$$

Here

$$\sqrt{p_1 p_2}(\ln 1/\sigma_1 + \ln 1/\sigma_2) = 10/27 > 1/e,$$

so that the condition (e) is satisfied. However,

$$p_1 \ln 1/\sigma_1 + p_2 \ln 1/\sigma_2 = 2/9 < 1/e,$$

and so (d) does not hold.

On the other hand, for the equation

$$x'(t) + \frac{1}{8et} x(e^{-1}t) + \frac{1}{2et} x(e^{-2}t) = 0, \quad t \geqq a > 0, \tag{15}$$

we have

$$p_1 \ln 1/\sigma_1 + p_2 \ln 1/\sigma_2 = 9/8e > 1/e,$$

so that the condition (d) holds. But

$$\sqrt{p_1 p_2}(\ln 1/\sigma_1 + \ln 1/\sigma_2) = 3/4e < 1/e,$$

and so (e) is not satisfied.

Remark 2. In the case $n = 1$ each of the conditions (a)—(e) reduces to $p \ln 1/\sigma > 1/e$ which is both a necessary and sufficient condition for oscillation of all the solutions of the equations

$$x'(t) + \frac{p}{t} x(\sigma t) = 0, \quad t \geqq a > 0 \tag{16}$$

and

$$x'(t) + \frac{p}{t \ln t} x(t^{\sigma}) = 0, \quad t \geqq a > 1, \tag{17}$$

where $p > 0$ and $0 < \sigma < 1$.

By similar arguments we can establish parallel results concerning the advanced equations

$$x'(t) - t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) = 0, \quad t \geqq a > 0 \tag{18}$$

and

$$x'(t) - (t \ln t)^{-1} \sum_{i=1}^{n} p_i x(t^{\sigma_i}) = 0, \quad t \geqq a > 1, \tag{19}$$

where $p_i > 0$, $\sigma_i \geqq 1$, $i = 1, 2, \ldots, n$, are constants.

Theorem 1′. A necessary and sufficient condition for all the solutions of Eq. (18) (or (19)) to be oscillatory is that

$$\alpha - \sum_{i=1}^{n} p_i \sigma_i^{\alpha} < 0 \tag{20}$$

for all $\alpha > 0$.

Theorem 2′. All the solutions of Eq. (18) (or (19)) oscillate if and only if

$$\alpha_0 - \sum_{i=1}^{n} p_i \sigma_i^{\alpha_0} < 0 \tag{21}$$

where α_0 is the unique solution of

$$\sum_{i=1}^{n} p_i (\ln \sigma_i) \sigma_i^{\alpha} = 1. \tag{22}$$

Theorem 3′. Each of the following conditions implies that every solution of Eq. (18) (or (19)) is oscillatory:

(a) $p_i \ln \sigma_i > 1/e$ for some $i \in \{1, 2, \ldots, n\}$;

(b) $\sum_{i=1}^{n} p_i \ln \sigma > 1/e$, where $\sigma = \min\{\sigma_i, i = 1, \ldots, n\}$;

(c) $n\Delta_s(p_1 \ln \sigma_1, \ldots, p_n \ln \sigma_n) > 1/e$ for some $s < 1$;

(d) $\sum_{i=1}^{n} p_i \ln \sigma_i > 1/e$;

(e) $\left(\prod_{i=1}^{n} p_i\right)^{1/n} \left(\sum_{i=1}^{n} \ln \sigma_i\right) > 1/e$.

Finally, we note that all the results presented in this paper remain valid if we replace Eqs. (1), (2), (18) and (19) by the retarded differential inequalities

$$\operatorname{sgn} x(t) \left\{ x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) \right\} \leqq 0, \tag{23}$$

$$\operatorname{sgn} x(t) \left\{ x'(t) + (t \ln t)^{-1} \sum_{i=1}^{n} p_i x(t^{\sigma_i}) \right\} \leqq 0, \tag{24}$$

where $p_i > 0$, $0 < \sigma_i \leqq 1$, $i = 1, 2, \ldots, n$, and the advanced inequalities

$$\operatorname{sgn} x(t) \left\{ x'(t) - t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) \right\} \geqq 0, \tag{25}$$

$$\operatorname{sgn} x(t) \left\{ x'(t) - (t \ln t)^{-1} \sum_{i=1}^{n} p_i x(t^{\sigma_i}) \right\} \geqq 0, \tag{26}$$

where $p_i > 0$, $\sigma_i \geqq 1$, $i = 1, 2, \ldots, n$, respectively.

We shall provide a brief outline of the proof of Theorem 1 for the inequality (23).

The necessity part is obvious. In order to prove the sufficiency part, let us assume that the inequality (23) has an eventually positive solution $x(t)$. Then, according to the comparison result of Philos [11, Corollary 1], the corresponding differential equation (1) has an eventually positive solution $u(t)$, which is a contradiction with the assertion of Theorem 1 applied to (1).

Similarly we can prove that the sufficiency parts of Theorems 1 and 2 (and, consequently, Theorem 3) remain true also for the nonlinear delay differential inequality

$$\operatorname{sgn} x(t)\{x'(t)+f(t, x(g_1(t)), \ldots, x(g_n(t)))\} \leqq 0, \tag{27}$$

where the following conditions are satisfied: there exist constants $p_i > 0$, $0 < \sigma_i \leqq 1$, $i = 1, 2, \ldots, n$, and $T \geqq a > 0$ such that either

(i) $$f(t, u_1, \ldots, u_n) \operatorname{sgn} u_1 \geqq t^{-1} \sum_{i=1}^{n} p_i |u_i|$$

for $t \geqq T$, $u_1 u_i > 0$, $i = 1, \ldots, n$, $\lim_{t \to \infty} g_i(t) = \infty$

$$\text{and } g_i(t) \leqq \sigma_i t \quad \text{for } t \geqq T, \quad i = 1, \ldots, n,$$

or

(ii) $$f(t, u_1, \ldots, u_n) \operatorname{sgn} u_1 \geqq (t \ln t)^{-1} \sum_{i=1}^{n} p_i |u_i|$$

for $t \geqq T$, $u_1 u_i > 0$, $i = 1, \ldots, n$, $\lim_{t \to \infty} g_i(t) = \infty$

$$\text{and } g_i(t) \leqq t^{\sigma_i} \quad \text{for } t \geqq T, \quad i = 1, \ldots, n.$$

We note that under the above assumptions the sufficiency part of Theorem 1 gives Corollaries 2 and 3 in [10].

The parallel results concerning the nonlinear advanced differential inequality

$$\operatorname{sgn} x(t)\{x'(t)-f(t, x(g_1(t)), \ldots, x(g_n(t)))\} \geqq 0, \tag{28}$$

can be also proved without much difficulty if we assume that there exist constants $p_i > 0$, $\sigma_i \geqq 1$, $i = 1, \ldots, n$, and $T \geqq a > 0$ such that either

(iii) $$f(t, u_1, \ldots, u_n) \operatorname{sgn} u_1 \geqq t^{-1} \sum_{i=1}^{n} p_i |u_i|$$

for $t \geqq T$, $u_1 u_i > 0$, $i = 1, \ldots, n$, and $g_i(t) \geqq \sigma_i t$
for $t \geqq T$, $i = 1, \ldots, n$,

or

(iv) $$f(t, u_1, \ldots, u_n) \operatorname{sgn} u_1 \geqq (t \ln t)^{-1} \sum_{i=1}^{n} p_i |u_i|$$

for $t \geqq T$, $u_1 u_i > 0$, $i = 1, \ldots, n$, and $g_i(t) \geqq t^{\sigma_i}$
for $t \geqq T$, $i = 1, \ldots, n$.

The details of this extension are left to the reader.

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SÚHRN

NUTNÉ A POSTAČUJÚCE PODMIENKY OSCILÁCIE DIFERENCIÁLNYCH RO/NÍC A NEROVNÍC PRVÉHO RÁDU S ONESKORENÝM ARGUMENTOM

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V práci sú odvodené nutné a postačujúce podmienky oscilácie všetkých riešení lineárnych diferenciálnych rovníc typu

$$x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) = 0 \tag{1}$$

a

$$y'(t) + (t \ln t)^{-1} \sum_{i=1}^{n} p_i y(t^{\sigma_i}) = 0, \tag{2}$$

kde $p_i > 0$ a $0 < \sigma_i \leqq 1$, $i = 1, 2, \ldots, n$, sú konštanty.

РЕЗЮМЕ

НЕОБХОДИМЫЕ И ДОСТАТОЧНЫЕ УСЛОВИЯ КОЛЕБЛЕМОСТИ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ И НЕРАВЕНСТВ ПЕРВОГО ПОРЯДКА С ЗАПАЗДЫВАЮЩИМ АРГУМЕНТОМ

Ярослав Ярош, Братислава

В работе приведены необходимые и достаточные условия колеблемости всех решений линейных дифференциальных уравнений вида

$$x'(t) + t^{-1} \sum_{i=1}^{n} p_i x(\sigma_i t) = 0 \tag{1}$$

и

$$y'(t) + (t \ln t)^{-1} \sum_{i=1}^{n} p_i y(t^{\sigma_i}) = 0, \tag{2}$$

где $p_i > 0$, $0 < \sigma_i \leqq 1$, $i = 1, \ldots, n$, постоянные.