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BOOLEAN MATRICES: A NOTE ON A PAPER BY D. OLEJÁR

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A stimulation to this note was given by a paper of D. Olejár [4] concerning the number of ones in almost all boolean matrices. More accurately the content of that paper is a proof of the following assertion:

For almost every boolean matrix $\mathbf{A}_N$ of order N the inequality

$$|N^2/2 - t(A_N)| < \eta(N)N \quad \text{holds,}$$

where $\eta(N) \to \infty$, $\eta^3(N) = o(N)$ and $t(\mathbf{A}_N)$ is the number of ones in $\mathbf{A}_N$.

Since the number of boolean matrices with exactly i ones is equal to $\binom{N^2}{i}$ and the number of all boolean matrices is equal to 2^{N^2}, the above problem reposes on finding the value of λ for which

$$\sum_{i>\lambda} \binom{N^2}{i} / 2^{N^2} \to 0, \quad \text{if } N \to \infty.$$

Thus, it suffices to know the behaviour of a binomial distribution in the tails. But at present this is already a notoriously known area of combinatorics having a long history. Since 1952, when H. Chernoff [1] proved his inequality

$$\sum_{i>k} \binom{n}{i} p^i q^{n-i} \leqslant \exp[(n-k)\log(nq/n-k) + k\log(np/k)], \tag{1}$$

where $k \geqslant pn$ and $p + q = 1$,
this inequality has been frequently used. For example J. W. Moon, see the well-known monography [2] by Erdös and Spencer or its Russian translation [3], sets $p = q = 1/2$, $k = n/2 + \lambda$ in (1) to show

$$\sum_{i>n/2+\lambda} \binom{n}{i} < 2^n \exp[-2\lambda^2/n], \tag{2}$$

given $0 \leqslant \lambda \leqslant n/2$.

Set $n = N^2$ and $\lambda = \omega(N)N$ in (2), where $\omega(N) \to \infty$.

Immediately we obtain even a strengthening of the theorem of D. Olejár [4], namely:

Theorem. For almost every boolean matrix $\mathbf{A}_N$ of order N the inequality

$$|N^2/2 - t(\mathbf{A}_N)| < \omega(N)N \quad \text{holds,}$$

where $\omega(N) \to \infty$ is an arbitrary slowly inereasing function and $t(A_N)$ is the number of ones in $\mathbf{A}_N$.

Remark. Analogously one can apply the above inequalities (1) and (2) to the results of D. Olejár in [5] which are concerned with almost all boolean matrices, for example to Theorem 2.1. in [5] and others.

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SÚHRN

BOOLEOVSKÉ MATICE: POZNÁMKA K PRÁCI D. OLEJÁRA

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Práca obsahuje krátky dôkaz istej teorémy D. Olejára týkajúcej sa skoro všetkých booleovských matíc získaný ako priamy dôsledok všeobecne známeho výsledku z kombinatoriky.

РЕЗЮМЕ

БУЛЕВСКИЕ МАТРИЦЫ: ЗАМЕЧАНИЕ К РАБОТЕ Д. ОЛЕЙАРА

Павел Томаста, Братислава

Работа содержит короткое доказательство одной теоремы Д. Олейара, касающейся почти всех булевских матриц, приобретен как прямое следствие общеизвестного комбинаторического результата.