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ON OSCILLATION OF CERTAIN CLASS OF SOLUTIONS OF RETARDED DIFFERENTIAL INEQUALITIES

JAROSLAV JAROŠ, Bratislava

The purpose of this paper is to extend and improve, in several directions, recent results due to Grace and Lalli [5] concerning the forced nonlinear retarded differential inequalities of the form

$$x(t)\left\{L_n x(t) + \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) - h(t)\right\} \leqq 0, \text{ for } n \text{ odd}, \tag{1}$$

and

$$x(t)\left\{L_n x(t) - \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) - h(t)\right\} \geqq 0, \text{ for } n \text{ even}, \tag{2}$$

where L_n is the general disconjugate differential operator defined by $L_0(x(t) = a_0(t)x(t)$ and

$$L_k x(t) = a_k(t)\frac{\mathrm{d}}{\mathrm{d}t}L_{k-1}x(t), \quad k = 1, 2, \ldots, n.$$

We shall assume that $a_i(t)$, $i = 0, 1, \ldots, n$, are positive and continuous functions on $[t_0, \infty)$ and the operator L_n is in canonical form in the sense that

$$\int_{t_0}^{\infty} a_i^{-1}(t)\,\mathrm{d}t = \infty \tag{3}$$

for $i = 1, 2, \ldots, n-1$. In what follows, the set of all real-valued functions $y(t)$ defined on $[t_y, \infty)$ and such that $L_i y(t)$, $i = 0, 1, \ldots, n$, exist and are continuous on $[t_y, \infty)$ will be denoted by $\mathscr{D}(L_n)$.

As usual, we restrict our considerations only to those solutions $x(t)$ of (1) (or (2)) which exist on some ray $[t_x, \infty)$, $t_x \geqq t_0$, and satisfy

$$\sup\{|x(s)| : s \geqq t\} > 0$$

for every $t \in [t_x, \infty)$. Such a solution is called oscillatory if it has arbitrarily large zeros in $[t_x, \infty)$ and it is called nonoscillatory otherwise.

In regard to the inequalities (1) and (2) the following conditions are assumed to hold:

(i) $p_i \in C([t_0, \infty), (0, \infty))$, $i = 1, \dots, m$;

(ii) $f_i \in C(R, R)$, $y f_i(y) > 0$ for $y \neq 0$, f_i are nondecreasing and $|f_i(xy)| \geqq |f_i(x) f_i(y)|$ for every x, y and $i = 1, \dots, m$;

(iii) $g_i \in C([t_0, \infty), R)$, $g_i(t) \leqq t$ for $t \geqq t_0$, $\lim g_i(t) = \infty$ as $t \to \infty$, g_i are nondecreasing on $[t_0, \infty)$, $i = 1, \dots, m$;

(iv) $h \in C([t_0, \infty), R)$ and there exists a function $p \in \mathscr{D}(L_n)$ such that $L_n p(t) = h(t)$ and $L_0 p(t)$ is strongly bounded on $[t_0, \infty)$ in the sense that for every $T \geqq t_0$ there are T_*, $T^* \geqq T$ such that

$$L_0 p(T_*) = \min_{t \in [T, \infty)} L_0 p(t) \text{ and } L_0 p(T^*) = \max_{t \in [T, \infty)} L_0 p(t).$$

To formulate our results we shall use the following notation. Let $j_r \in \{1, \dots, n-1\}$, $r = 1, 2, \dots, n-1$ and $t, s \in [t_0, \infty)$. We define $I_0 = 1$ and

$$I_r(t, s; j_1, \dots, j_r) = \int_s^t a_{j_1}^{-1}(\tau) I_{r-1}(\tau, s, j_2, \dots j_r) \, d\tau.$$

It is not difficult to verify that for $1 \leqq r \leqq n-1$

$$I_r(t, s; j_1, \dots, j_r) = (-1)^r I_r(s, t; j_r, \dots, j_1) \tag{4}$$

and for $i = 1, 2, \dots, r$

$$I_r(t, s; j_1, \dots, j_r) = \\ = \int_s^t I_{i-1}(t, \tau; j_1, \dots, j_{i-1}) a_{j_i}^{-1}(\tau) I_{r-i}(\tau, s; j_{i+1}, \dots, j_r) \, d\tau. \tag{5}$$

Moreover, if $y \in \mathscr{D}(L_n)$, $0 \leqq i \leqq r \leqq n-1$ and $t, s \in [t_x, \infty)$, then we can easily derive the following generalization of the Taylor's formula:

$$L_i y(t) = \sum_{j=i}^{r} (-1)^{j-i} L_j y(s) I_{j-i}(s, t; j, \dots, i+1) + \\ + (-1)^{r-i+1} \int_t^s I_{r-i}(\tau, t; r, \dots, i+1) \frac{L_{r+1} y(\tau)}{a_{r+1}(\tau)} \, d\tau \tag{6}$$

(see for example Chanturija [1]). Using (4), the above formula can be rewritten as

$$L_i y(t) = \sum_{j=i}^{r} L_j y(s) I_{j-i}(t, s; i+1, \dots, j) + \tag{7}$$

$$+\int_s^t I_{r-i}(t, \tau; i+1, \ldots, r)\frac{L_{r+1}y(\tau)}{a_{r+1}(\tau)}\mathrm{d}\tau,$$

$i = 0, 1, \ldots, r$; $r = 0, 1, \ldots, n-1$.

For simplicity we shall frequently use the abbreviations:

$$\alpha_k(t, s) = a_0^{-1}(t)I_k(t, s; 1, \ldots, k), \qquad \alpha_k(t) = \alpha_k(t, t_0),$$

$$\omega_k(t, s) = a_n^{-1}(t)I_k(t, s; n-1, \ldots, n-k), \quad \omega_k(t) = \omega_k(t, t_0).$$

Now let $y \in \mathscr{D}(L_n)$ be such that

$$(-1)^n y(t)L_n y(t) > 0$$

for all sufficiently large t. Then according to a generalization of a well-known lemma of Kiguradze (see [18, Lemma 2]), there exist an even integer ℓ, $0 \leqq \ell \leqq n$, and a $t_1 \geqq t_0$ such that

$$y(t)L_i y(t) > 0 \text{ on } [t_1, \infty) \text{ for } i = 0, 1, \ldots, l, \tag{8}$$

and

$$(-1)^{i-\ell} y(t)L_i y(t) > 0 \text{ on } [t_1, \infty) \text{ for } i = \ell, \ell+1, \ldots, n. \tag{9}$$

Such a $y(t)$ is said to be a (nonoscillatory) function of degree ℓ (see Foster and Grimmer [3] and Kusano, Naito and Tanaka [13]).

It is not difficult to verify that the following extension of a result due to Grace and Lalli [4, Theorem 2] holds. The proof can be modelled on that of Theorem 1 in [4], so we omit the details.

Lemma. Suppose that $h(t) = 0$ and that $k \in \{1, 2, \ldots, n-1\}$ is fixed. Let $x(t)$ be a nonoscillatory solution of (1) (or (2)) such that

$$\lim_{t\to\infty}\frac{x(t)}{\alpha_k(t)} = 0.$$

Then there exist an even integer ℓ, $0 \leqq \ell \leqq k$, and a $t_1 \geqq t_0$ such that $x(t)$ is the function of degree ℓ on $[t_1, \infty)$.

Remark 1. In [4], Grace and Lalli stated this lemma for $k = 1$ and required moreover the satisfaction of the condition

$$\lim_{t\to\infty}\frac{1}{\alpha_1(t)}\sum_{i=0}^{v} c_i\alpha_i(t) > 0$$

for every choice of the constants c_i with $c_v > 0$, $v = 1, 2, \ldots, n-1$. We note that if (3) holds, then the above condition as well as the more general condition

$$\lim_{t\to\infty}\frac{1}{\alpha_k(t)}\sum_{i=k}^{v} c_i\alpha_i(t) > 0, \quad 1 \leqq k \leqq n-1,$$

for every choice of c_i with $c_\nu > 0$, $\nu = k, k+1, \dots, n-1$, are always satisfied (see [7]).

Let $f(x) = \min_{1 \le i \le m} f_i(x)$ and $g(t) = \max_{1 \le i \le m} g_i(t)$.

Theorem 1 (Unforced Oscillation). Assume that $h(t) = 0$ and that $k \in \in \{1, 2, \dots, n-1\}$ is fixed. If

$$\limsup_{t \to \infty} \int_{g(t)}^{t} \omega_{n-\ell-1}(\tau, g(t)) \sum_{i=1}^{m} p_i(\tau) f_i(\alpha_\ell(g_i(\tau), g_i(t)))\, d\tau > \\ > \limsup_{z \to 0} \frac{z}{f(z)} \tag{10}$$

fo every $\ell = 0, 2, \dots, k$, if k is even, and $\ell = 0, 2, \dots, k-1$, if k is odd, then every solution $x(t)$ of (1) (or (2)) such that

$$\lim_{t \to \infty} \frac{x(t)}{\alpha_k(t)} = 0$$

is oscillatory.

Proof. In order to avoid repetition, we consider only the inequality (1).

Assume to the contrary that there exists a nonoscillatory solution $x(t)$ of (1) such that $x(t)/\alpha_k(t) \to 0$ as $t \to \infty$. By our Lemma, there exist an even integer ℓ, $0 \leqq \ell \leqq k$, and a $t_1 \geqq t_0$ such that $x(t)$ is of degree ℓ on $[t_1, \infty)$, i.e. (8) and (9) hold. Suppose first that $x(t) > 0$ for $t \geqq t_1$ and choose $T \geqq t_1$ such that $g_i(t) \geqq t_1$ for $t \geqq T$ and $i = 1, \dots, m$. From the formula (6) applied to $x(t)$ with i, r, s and t replaced by ℓ, $n-1$, t and $g(t)$, respectively, and from (9) it follows that

$$L_\ell x(g(t)) = \sum_{j=\ell}^{n-1} (-1)^{j-\ell} L_j x(t) I_{j-\ell}(t, g(t); j, \dots, \ell+1) + \\ + (-1)^{n-\ell} \int_{g(t)}^{t} I_{n-\ell-1}(\tau, g(t); n-1, \dots, \ell+1) \frac{L_n x(\tau)}{a_n(\tau)} d\tau \\ \geqq L_\ell x(t) - \int_{g(t)}^{t} \omega_{n-\ell-1}(\tau, g(t)) L_n x(\tau)\, d\tau$$

for $t \geqq T$. Thus, taking (1) into account, we have

$$L_\ell x(g(t)) \geqq L_\ell x(t) + \int_{g(t)}^{t} \omega_{n-\ell-1}(\tau, g(t)) \times \\ \times \sum_{i=1}^{m} p_i(\tau) f_i(x(g_i(\tau)))\, d\tau \tag{11}$$

for $t \geqq T$.

Suppose first that $\ell = 0$. Then, by (9) and (iii), $L_0x(g(t))$ is nonincreasing on $[T, \infty)$ and from (11) we get

$$L_0x(g(t)) \geqq L_0x(t) + \int_{g(t)}^{t} \omega_{n-1}(\tau, g(t)) \times$$
$$\times \sum_{i=1}^{m} p_i(\tau) f_i(L_0x(g_i(\tau))) f_i(a_0^{-1}(g_i(\tau)))\,\mathrm{d}\tau \geqq \tag{12}$$
$$\geqq L_0x(t) + f(L_0x(g(t))) \int_{g(t)}^{t} \omega_{n-1}(\tau, g(t)) \sum_{i=1}^{m} p_i(\tau) f_i(a_0^{-1}(g_i(\tau)))\,\mathrm{d}\tau.$$

Since $L_1x(t) < 0$ for $t \geqq t_1$, $L_0x(t)$ decreases to a limit $c \geqq 0$ as $t \to \infty$. From (12) we obtain $c = 0$.

Thus

$$\limsup_{t\to\infty} \frac{L_0x(g(t))}{f(L_0x(g(t)))} \geqq \limsup_{t\to\infty} \int_{g(t)}^{t} \omega_{n-1}(\tau, g(t)) \sum_{i=1}^{m} p_i(\tau) f_i(a_0^{-1}(g_i(\tau)))\,\mathrm{d}\tau,$$

a contradiction to (10) in the case $\ell = 0$.

Now let $\ell > 0$. Application of the formula (7) to the case where $y(t)$, i, r, t and s are replaced by $x(t)$, 0, $\ell - 1$, τ and t, respectively, shows that

$$L_0x(\tau) = \sum_{j=0}^{\ell-1} L_jx(t) I_j(\tau, t; 1, \dots, j) +$$
$$+ \int_t^{\tau} I_{\ell-1}(\tau, s; 1, \dots, \ell - 1) a_\ell^{-1}(s) L_\ell x(s)\,\mathrm{d}s \geqq$$
$$\geqq \int_t^{\tau} I_{\ell-1}(\tau, s; 1, \dots, \ell - 1) a_\ell^{-1}(s) L_\ell x(s)\,\mathrm{d}s \geqq$$
$$\geqq L_\ell x(\tau) \int_t^{\tau} I_{\ell-1}(\tau, s; 1, \dots, \ell - 1) a_\ell^{-1}(s)\,\mathrm{d}s =$$
$$= L_\ell x(\tau) I_\ell(\tau, t; 1, \dots, \ell), \qquad \tau \geqq t \geqq t_1,$$

where we have used (8), the decreasing character of $L_\ell x(t)$ on $[t_1, \infty)$ and the identity (5) for $i = r = \ell$.

Since the functions $g_i(t)$, $i = 1, 2, \dots, m$, are nondecreasing for $t \geqq t_0$, from the above we get

$$L_0x(g_i(\tau)) \geqq L_\ell x(g_i(\tau)) I_\ell(g_i(\tau), g_i(t); 1, \dots, \ell),$$

i.e.

$$x(g_i(\tau)) \geqq L_\ell x(g_i(\tau)) \alpha_\ell(g_i(\tau), g_i(t)),$$

for $\tau \geqq t \geqq T$ and $i = 1, 2, \dots, m$.

Using this estimation in (11), taking into account (ii) and the fact that the function $L_\ell x(t)$ is decreasing on $[t_1, \infty)$, we obtain

$$L_\ell x(g(t)) \geqq L_\ell x(t) + f(L_\ell x(g(t))) \int_{g(t)}^{t} \omega_{n-\ell-1}(\tau, g(t)) \times \\ \times \sum_{i=1}^{m} p_i(\tau) f_i(\alpha_\ell(g_i(\tau), g_i(t)))\, d\tau \tag{13}$$

for $t \geqq T$. From (13) it follows as in the case $\ell = 0$ that $\lim L_\ell x(t) = 0$ as $t \to \infty$. Finally, dividing both sides of the above inequality by $f(L_\ell x(g(t)))$ and taking the limit superior as $t \to \infty$, we get again the contradiction to (10).

A similar argument holds for $x(t)$ eventually negative and this completes the proof.

Remark 2. It follows from the proof of the above theorem that in the case $k = 1$ we can avoid the condition

$$|f_i(xy)| \geqq |f_i(x) f_i(y)| \text{ for all } x, y \text{ and } i = 1, \dots, m,$$

and (10) becomes

$$\limsup_{t \to \infty} \int_{g(t)}^{t} \omega_{n-1}(\tau, g(t)) \sum_{i=1}^{m} p_i(\tau)\, d\tau > \limsup_{z \to 0} \frac{z}{f(z)}.$$

In the spirit of this remark our Theorem 1 generalizes Theorem 1 in [5].

Corollary 1. Assume that $a_0(t) = a_1(t) = \dots = a_n(t) = 1$, $h(t) = 0$ and $k \in \{1, 2, \dots, n-1\}$ is fixed. If

$$\limsup_{t \to \infty} \int_{g(t)}^{t} (\tau - g(t))^{n-\ell-1} \sum_{i=1}^{m} p_i(\tau) f_i((g_i(\tau) - g_i(t))^\ell)\, d\tau > \\ > (n - \ell - 1)! \limsup_{t \to \infty} \frac{z}{f(z)} \tag{14}$$

for every $\ell = 0, 2, \dots, k$, if k is even, and $\ell = 0, 2, \dots, k-1$ if k is odd, then every solution $x(t)$ of (1) (or (2)) such that

$$\lim_{t \to \infty} \frac{x(t)}{t^k} = 0$$

is oscillatory.

In our next result we shall show that Theorem 3 in [5] remains valid for more general forcing functions than those considered in [5], namely, for the functions $h(t)$ which satisfy (iv). For this purpose denote

$$p_*(t) = \min_{\tau \in [t, \infty)} L_0 p(\tau),$$

$$p^*(t) = \max_{\tau \in [t, \infty)} L_0 p(\tau),$$

$$p_1 = \lim_{t \to \infty} p_*(t),$$

$$p_2 = \lim_{t \to \infty} p^*(t).$$

Theorem 2. (Forced Oscillation) Suppose that

$$\limsup_{t \to \infty} a_0(t) < \infty \tag{15}$$

and

$$\limsup_{t \to \infty} \int_{g(t)}^{t} \omega_{n-1}(s, g(t)) \sum_{i=1}^{m} p_i(s)\, \mathrm{d}s > M, \tag{16}$$

where M is a positive constant. Then every solution $x(t)$ of (1) (or (2)) such that

$$\lim_{t \to \infty} \frac{x(t)}{\alpha_1(t)} = 0$$

is either oscillatory or

$$\lim_{t \to \infty} [L_0 x(t) - L_0 p(t)] = -p_1 \text{ or } -p_2.$$

Proof. We consider only (1).

Let $x(t)$ be a solution of (1) such that $\lim_{t \to \infty} (x(t)/\alpha_1(t)) = 0$. Assume that this solution is positive for $t \geqq t_1 \geqq t_0$. By (iii), we choose $t_2 \geqq t_1$ such that $x(g_i(t)) > 0$ for $t \geqq t_2$ and $i = 1, 2, \ldots, m$. Put $u(t) = x(t) - p(t)$. Then, in view of (1) and (iv), we obtain

$$L_n u(t) + \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) \leqq 0, \; t \geqq t_2, \tag{17}$$

which implies that $L_n u(t) < 0$ for $t \geqq t_2$.

It can be easily verified (as in the prooof of Theorem 1 in [4]) that there exists $t_3 \geqq t_2$ such that for $t \geqq t_3$ and $k = 1, 2, \ldots, n$

$$(-1)^k L_k u(t) > 0. \tag{18}$$

In particular, $L_1 u(t) < 0$ on $[t_3, \infty)$ and so $\lim_{t \to \infty} L_0 u(t) = c$ where c is a constant.

Put $z(t) = L_0 u(t) + p_*(t)$. Then we have

$$\lim_{t \to \infty} z(t) = \lim_{t \to \infty} [L_0 u(t) + p_*(t)] = c + p_1 = d.$$

If $d < 0$, then $L_0u(t) + p_*(t) < 0$ for sufficiently large t, say $t \geqq T \geqq t_3$. By (iv), there exists a $T_1 \geqq T$ such that

$$L_0u(T_1) + p_*(T_1) = L_0u(T_1) + L_0p(T_1) =$$
$$= L_0x(T_1) - L_0p(T_1) + L_0pT_1) =$$
$$= L_0x(T_1) > 0,$$

a contradiction.

If $d > 0$, then we have

$$L_0x(t) = L_0u(t) + L_0p(t) \geqq L_0u(t) + p_*(t) = z(t) > \frac{d}{2}$$

for sufficiently large t, and so, by (15) and (iii),

$$x(g_i(t)) > k$$

for some constant $k > 0$, $i = 1, 2, \ldots, m$ and every large t.

From (17) we obtain

$$L_nu(t) + \sum_{i=1}^{m} p_i(t)f(k) \leqq 0. \tag{19}$$

Next, from the formula (6) applied to $u(t)$ with i, r, s and t replaced by $0, n-1$, t and $g(t)$, respectively, and from (18) we get

$$\begin{aligned} L_0u(g(t)) &= L_0u(t) + \sum_{j=1}^{n-1} (-1)^j L_ju(t)I_j(t, g(t); j, \ldots, 1) - \\ &- \int_{g(t)}^{t} I_{n-1}(\tau, g(t); n-1, \ldots, 1)\frac{L_nu(\tau)}{a_n(\tau)}\mathrm{d}\tau \geqq \\ &\geqq L_0u(t) - \int_{g(t)}^{t} I_{n-1}(\tau, g(t); n-1, \ldots, 1)\frac{L_nu(\tau)}{a_n(\tau)}\mathrm{d}\tau \end{aligned} \tag{20}$$

for sufficiently large t. Combining (19) and (20) we obtain

$$L_0u(g(t)) \geqq L_0u(t) + f(k)\int_{gt)}^{t} \omega_{n-1}(\tau, g(t); n-1, \ldots, 1)\sum_{i=1}^{m} p_i(\tau)\mathrm{d}\tau.$$

Finally, taking the limit superior as $t \to \infty$, we obtain the contradiction to (16). Thus, we conclude that $d = 0$, which implies

$$\lim_{t\to\infty} [L_0x(t) - L_0p(t)] = \lim_{t\to\infty} [z(t) - p_*(t)] = -p_1.$$

A parallel argument holds if we assume that (1) has a negative solution $x(t)$ such that $x(t)/\alpha_1(t) \to 0$ as $t \to \infty$. In this case we prove that

$$\lim_{t \to \infty} [L_0 x(t) - L_0 p(t)] = -p_2 .$$

This completes the proof.

Corollary 2. Let $a_0(t) = a_1(t) = \dots = a_n(t) = 1$. If

$$\limsup_{t \to \infty} \int_{g(t)}^{t} (s - g(t))^{n-1} \sum_{i=1}^{m} p_i(s)\, ds > M ,$$

where M is a positive constant, then every solution $x(t)$ of (1) (or (2)) such that

$$\lim_{t \to \infty} \frac{x(t)}{t} = 0$$

is either oscillatory or

$$\lim_{t \to \infty} [x(t) - p(t)] = -p_1 \text{ or } -p_2 .$$

Example. The second order retarded linear equation

$$x''(t) - t^{-2}[e^{-3\pi/2} x(e^{-3\pi/2} t) + e^{-\pi/2} x(e^{-\pi/2} t)] = \frac{\sin(\log t) - 3\cos(\log t)}{t^3}, \tag{21}$$

$t \geqq 1$, satisfies all the conditions of Corollary 2 with $p(t) = 1 + \sin(\log t)/t$. Thus every solution $x(t)$ of (21) such that $x(t)/t \to 0$ as $t \to \infty$ is either oscillatory or

$$\lim_{t \to \infty} [x(t) - 1 - \sin(\log t)/t] = -1 .$$

In fact, $x(t) = \dfrac{2 + \sin(\log t)}{t}$ is one such (nonoscillatory) solution.

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Author's address:
Jaroslav Jaroš
Katedra matematickej analýzy MFF UK
Mlynská dolina
842 15 Bratislava

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SÚHRN

O OSCILÁCII URČITEJ TRIEDY RIEŠENÍ RETARDOVANÝCH DIFERENCIÁLNYCH NEROVNÍC

Jaroslav Jaroš, Bratislava

V práci sú dokázané kritéria oscilatoričnosti určitých špeciálnych riešení nelineárnych diferenciálnych nerovníc s oneskoreným argumentom

$$x(t)\left\{L_n x(t) + \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) - h(t)\right\} \leqq 0, \; n \text{ nepárne},$$

$$x(t)\left\{L_n x(t) - \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) - h(t)\right\} \geqq 0, \; n \text{ párne},$$

kde L_n je zovšeobecnený diskonjugovaný diferenciálny operátor a $h(t)$ reprezentuje silne ohraničenú nútiacu funkciu.

РЕЗЮМЕ

О КОЛЕБЛЕМОСТИ НЕКОТОРОГО КЛАССА РЕШЕНИЙ ДИФФЕРЕНЦИАЛЬНЫХ НЕРАВЕНСТВ С ЗАПАЗДЫВАЮЩИМ АРГУМЕНТОМ

Ярослав Ярош, Братислава

В работе доказаны признаки колеблемости некоторых специальных решений нелинейных дифференциальных неравенств с запаздывающим аргументом

$$x(t)\left\{L_n x(t) + \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) - h(t)\right\} \leqq 0, \; n \text{ нечетное},$$

$$x(t)\left\{L_n x(t) - \sum_{i=1}^{m} p_i(t) f_i(x(g_i(t))) - h(t)\right\} \geqq 0, \; n \text{ четное},$$

где L_n обобщенный осцилляционный дифференциальный оператор и $h(t)$ представляет сильно ограниченную вынуждающую функцию.