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THE DECOMPOSITION OF AN INTERVAL OF NATURAL NUMBERS INTO FOUR STRONGLY SUM-FREE SETS (PRELIMINARY ANNOUNCEMENT)

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A set X of positive integers is the so-called strongly sum-free set if there do not exist $x, y, z \in X$ such that

$$x + y = z, \quad x \neq y.$$

Let us denote

$$[n, N] = \{n, n + 1, n + 2, \dots, N\}$$

an interval of positive integers.

In [1] there is proved that $[n, 5n + 2]$ for $n \geqq 2$ is the longest interval which is decomposible into two strongly sum-free sets. The decomposition has the form

$$A_0 = [n, 2n] \cup [4n + 3, 5n + 2]$$

$$B_0 = [2n + 1, 4n + 2].$$

For a decomposition into three strongly sum-free sets the interval $[n, 14n + 7]$, $n \geqq 2$ is the longest one which is the main result of [2] (see also [3]). For example, one of these decomposition is

$$A_1 = A_0 \cup [10n + 7, 11n + 6] \cup [13n + 8, 14n + 7]$$

$$B_1 = B_0 \cup [11n + 7, 13n + 7]$$

$$C_1 = [5n + 3, 10n + 6].$$

In [3] the author conjectured that $[n, 41n + 21]$, $n \geqq 2$ is longest for a decomposition on four strongly sum-free sets. One of possible decompositions is

$$A_2 = A_1 \cup [28n + 17, 29n + 16] \cup [31n + 18, 32n + 17] \cup$$
$$\cup [37n + 21, 38n + 2] \cup [40n + 22, 41n + 21]$$

$$B_2 = B_1 \cup [29n + 17, 31n + 17] \cup [38n + 21, 41n + 21]$$

$$C_2 = C_1 \cup [32n + 18, 37n + 20]$$

$$D_2 = [14n + 18, 28n + 16]$$

The following theorem is the first step in the proof of this conjecture.

Theorem. Let A, B, C, D be a decomposition of an interval $[n, N]$ on four strongly sum-free sets which satisfy $A_1 \subset A$, $B_1 \subset B$, and $C_1 \subset C$. Then $N \leqq 41n + 21$.

A proof can be done by contradiction. Let $N \geqq 41n + 22$. We have four possible cases: (i) $41n + 22 \in A$, (ii) $41n + 22 \in B$, (iii) $41n + 22 \in C$, (iv) $41n + 22 \in D$. Let us take out suitable sets $X_0 \subset A$, $Y_0 \subset B$, $Z_0 \subset C$, $U_0 \subset D$, and let us construct a quadruplet (X_0, Y_0, Z_0, U_0) from them. We can select two numbers from one of the sets X_0, Y_0, Z_0, U_0, e.g. $x, y \in X_0$ and we can construct $x + y$, if (j) $x \neq y$ and $x + y \leqq N$, or $x - y$, if (jj) $x \neq 2y$ and $x \geqq y + n$. Then $x \pm y \notin A$, therefore $x \pm y \in B$ or $x \pm y \in C$ or $x \pm y \in D$. Note that it need not be admissible for all of these cases. If it is allowed we add $x \pm y$ into Y_0 and we have a new quadruplet (X_0, Y_1, Z_0, U_0) and also we add $x \pm y$ into Z_0 and we have (X_0, Y_0, Z_1, U_0) and also we add $x \pm y$ into U_0 and we have (X_0, Y_0, Z_0, U_1). By repeating the forepoing procedure we get a tree where (X_i, Y_j, Z_k, U_l) are verteces. For all cases (i)—(iv) we can find a root of tree (Y_0, Y_0, Z_0, U_0) such that the corresponding tree ends by leaves $(X_{i0}, Y_{j0}, Z_{k0}, U_{l0})$ such that for every of them there exists a number $n' \leqq 41n + 21$ for which

$$n' = a \pm b = c \pm d = e \pm f = g \pm h$$

where $a, b \in X_{i0}$, $c, d \in Y_{j0}$, $e, f \in Z_{k0}$, $g, h \in U_{l0}$ and a, b, c, d, e, f, g, h satisfy corresponding inequalities (j), (jj) ((j if + and (jj) if −). This is a contradiction.

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SÚHRN

ROZKLAD INTERVALU PRIRODZENÝCH ČÍSEL NA ŠTYRI OSTRO SUMOVO-RIEDKE MNOŽINY

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Množina je ostro sumovo-riedka ak v nej nie je riešiteľná rovnica $x + y = z$, $x \neq y$. V práci je oznámený výsledok, podľa ktorého najdlhší interval celých kladných čísel, ktorý sa dá rozložiť na štyri takého množiny je $[n, 41n + 21]$ ($n \geqq 2$) za predpokladu, že tri z nich tvoria rozklad intervalu $[n, 14n + 7]$.

РЕЗЮМЕ

РАЗБИЕНИЕ ПРОМЕЖУТКА НАТУРАЛЬНЫХ ЧИСЕЛ НА ЧЕТЫРЕ СТРОГО СУММОВО-РЕДКИЕ МНОЖЕСТВА

Юлиус Бачик, Нитра

Множество называется строго суммого редким, если на нем не имеет решения уравнение $x + y = z$, $x \neq y$ В работе сформулирован результат, согласно которому целочисленый промежуток нейбольшей длины, который можно разбить на четыре строго суммого-редкие множества, есть $[n, 41n + 21]$, ($n \geqq 2$), в предположении, что три из ницх определяют разбинение промежутка n, $14n + 7$].