UNIVERSITAS COMENIANA
ACTA MATHEMATICA UNIVERSITATIS COMENIANAE
L—LI—1987

FUNCTIONS THAT PRESERVE UNIFOM DISTRIBUTION (PRELIMINARY ANNOUNCEMENT)

ŠTEFAN PORUBSKÝ, Bratislava

This is a preliminary report on a joint work with T. Šalát and O. Strauch and a more detailed version will be published elsewhere.

We shall use the following basic notation:
I stands for the interval $\langle 0, 1\rangle$,
M_I denotes the system of the functions from I to I,
R_I is the system of the Riemann-integrable functions from M_I.

We shall study properties of functions $f \in M_I$ sharing the property that for every uniformly distributed sequence $\{x_n\}_{n=1}^{\infty}$ of numbers from I also that sequence $\{f(x_n)\}_{n=1}^{\infty}$ is uniformly distributed in I. We shall denote by T_I the system of all such functions.

The starting result is the following one:

Theorem 1. A function $f \in T_I$ belongs to T_I if and only if for every function $g \in R_I$ also the composition $g \circ f$ belongs to R_I and

$$\int_0^1 g(x)\,\mathrm{d}x = \int_0^1 g(f(x))\,\mathrm{d}x .$$

The same proof technique leads to the expected modification:

Theorem 1′. A function $f \in M_I$ belongs to T_I if and only if for every $g \in C_I$ also $g \circ f \in R_I$ and

$$\int_0^1 g(x)\,\mathrm{d}x = \int_0^1 g(f(x))\,\mathrm{d}x .$$

The function $g(x) = x$ gives that every function in T_I is Riemann-integrable. However for such functions the following surprising result can be proved.

Theorem 2. Let $f \in R_I$. Then f belongs to T_I if there exists a uniformly distributed sequence $\{x_n\}_{n=1}^{\infty}$ in I such that $\{f(x_n)\}_{n=1}^{\infty}$ is also uniformly distributed.

The function

$$f(x) = \begin{cases} x \text{ if } x \text{ is rational} \\ 0 \text{ otherwise} \end{cases}$$

shows that there are functions $f \in R_I$ which preserve infinitely many uniformly distributed sequences and simultaneously for infinitely many not uniformly distributed sequences $\{x_n\}_{n=1}^{\infty}$ the sequence $\{f(x_n)\}_{n=1}^{\infty}$ is not uniformly distributed.

Some additional conditions on functions f from T_I reduce considerably the possibilities for f. Here are some of them:

Theorem 3. Let $f \in T_I$. If f is continuous and injective then either $f(x) = x$ or $f(x) = 1 - x$.

Theorem 4. If a function $f \in T_I$ has the derivative at every point interior to I then either $f(x) = x$ or $f(x) = 1 - x$.

Theorem 5. A Darboux function $f \in M_I$ belongs to T_I if and only if for every $x, x' \in I$ we have

$$|f(x) - f(x')| = |x' - x|.$$

A sufficient condition is given in the next result.

Theorem 6. If a function $f \in M_I$ has the property that

$$\max_{n \to \infty} (f(x_n) - x_n) = 0$$

for every sequence $\{x_n\}_{n=1}^{\infty}$ of numbers from I then $f \in T_I$.

There is a close connection between the functions $f \in R_I$ and the functions measurable in the Jordan sense. In this direction the following result can be proved.

Theorem 7. A function $f \in M_I$ belongs to T_I if and only if f is measurable in the Jordan sense and if

$$|f^{-1}(E)| = |E|$$

for every interval $E \subset I$.

This theorem enables us to construct examples of functions from T_I. The most simple of them are the zigzag functions with the height of every tooth exactly equal to one.

The closet generalization of zigzag functions are the piecewise linear functions. Let f be such a function. Let

$$f^{-1}(y) = \{x_1, x_2, \ldots, x_k\}$$

be the so called level set at $y \in i$. The function f has the derivative at every point of the level set for all but a finite set of points $y \in I$. It can be proved for instance:

Theorem 8. A piecewise linear function f belongs to T_I if and only if

$$\sum_{x_i \in f^{-1}(y)} \frac{1}{|f'(x_i)|} = 1$$

for every such $y \in I$ for which all the required derivatives exist.

This result in a rewritten form can be used for a construction of the all piecewise linear functions from T_I.

If we endow the set M_I with the supremum metric then it can be easily seen that the set T_I is closed in M_I. The topological properties of T_I in M_I culminates in the next theorem.

Theorem 9. The set T_I is perfect and nowhere dense in M_I.

The last result we mention is of a more specific nature. We first recall the notion of the somewhat continuous function. A function f from the topological space X to a topological space Y is somewhat continous if for every open set V in Y the condition $f^{-1}(V) \neq \emptyset$ implies that the interior $\mathrm{Int} f^{-1}(V)$ is also nonempty. Then Theorem 7 implies in turn that:

Theorem 10. Every function f from T_I is somewhat continuous.

Author's address:
Štefan Porubský
Matematický ústav SAV
ul. Obrancov mieru 49
814 73 Bratislava
Czechoslovakia

Received: 5. 1. 1986

SÚHRN

FUNKCIE ZACHOVÁVAJÚCE ROVNOMERNÉ ROZDELENIE

Štefan Porubský, Bratislava

V práci sa podávajú vybrané výsledky o funkciách $f: \langle 0, 1\rangle \mapsto \langle 0, 1\rangle$, ktoré prevádzajú každú rovnomerne rozdelenú postupnosť $\{x_n\}_{n=1}^{\infty}$ do rovnomerne rozdelenej postupnosti $\{f(x_n)\}_{n=1}^{\infty}$.

РЕЗЮМЕ

ФУНКЦИИ СОХРАНЯЮЩИЕ РАВНОМЕРНОЕ РАСПРЕДЕЛЕНИЕ

Штефан Порубски, Братислава

Описываются избранные результаты касающеися функций f из единичного интервала $\langle 0, 1\rangle$ в единичный интервал, которые переводят каждую равномерно распределенную последовательность $\{x_n\}_{n=1}^{\infty}$ в $\langle 0, 1\rangle$ в равномерно распределенную последовательность $\{f(x_n)\}_{n=1}^{\infty}$.

UNIVERSITAS COMENIANA
ACTA MATHEMATICA UNIVERSITATIS COMENIANAE
L—LI—1987

ON DISTRIBUTION FUNCTIONS OF SEQUECES

MILAN PAŠTÉKA, Bratislava

1. Introduction

The work has 3 parts. Part 1 is an introduction. In Part 2 we examine the topological properties of the metric space (M, d) from the point of view of distribution functions. Let (M, d) denote the metric space of all sequences of elements from interval $[0, 1)$ with the sup — metric d. In Part 3 we examine the transformation of sequences which preserve the distribution functions of sequences.

In this paper we used the following notation:

(i) $y = (y_n)$ is a sequence of elements of the interval $(0, 1]$.

(ii) y^k is a sequence of elements of M, where $y^k = (y_n^k)$.

For an interval $I \subset [0, 1]$, a sequence $y \in M$ and a positive integer N we used the standard notation the monograph [1].

(iii)
$$A(I, y, N) = \sum_{\substack{n \leq N \\ y_n \in I}} 1$$

(iv)
$$F_N(x) = \frac{A([0, x), y, N)}{N}$$

In [1] the notions of the asymptotic distribution function (ADF) and the distribution function (DF) of sequences are introduced.

Definition 1. Let $y \in M$ and g: $[0, 1] \to [0, 1]$. Then

(v) g is said to be an ADF of y if

$$F_N(x) \to g(x), \quad N \to \infty$$

for all $x \in [0, 1]$.

(vi) g is said to be a DF of y if there exists a sequence of indices (N_j) such that

$$F_{N_j}(x) \to g(x), \quad j \to \infty$$

for all $x \in [0, 1]$.

By investigation of the topological properties of (M, d) from the point of view of distribution functions we shall need the foolowing generalization of an ADF.
(vii) if a function g is nondecreasing on $[0, 1]$ and $g(0) = 0$, $g(1) = 1$, then g is said to be a weak asymptotic distribution function of y (abbreviated as WADF) if

$$F_N \xrightarrow{w} g$$

where $F_N \xrightarrow{w} g$ denotes that the sequence $(F_N(x))$ converges to $g(x)$ at each continuity point of g.

Every nondecreasing function g: $[0, 1] \to [0, 1]$ such that $g(0) = 0$, $g((1) = 1$ will be called a distribution function (DF).

Proposition 1.

(a′) Let a g be a DF. Then there exists a sequence $y \in M$ such that g is and ADF of y.

(a″) If g is DF of y and it is not ADF of y, then g has at least two DF.

(b) If y has increasing DF, then the sequence y is dense in $[0, 1]$.

(c) Let g be a DF of y. Then every point of discontinuity of g is a limit point y.

(d) If g is a continuous DF, then every sequence dense in $[0, 1]$ can be rearranged into a sequence whose ADF is g.

(e) Let g be a DF and $y \in M$. Then g is a WADF for the sequence y if and only if for each continuous function h: $[0, 1] \to R$ the function g and the sequence y satisfy the condition

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n) = \int_0^1 h dg .$$

The proofs af (a′), (e) can be found in [1], p. 53, the proof of (d) is in [2]. The proofs (a), (b), (c), are trivial.

2. Some topological properties of the space M

The metric d is defined in the following way:

$$d(y^1, y^2) = \sup \{|y_n^1 - y_n^2|; \quad n = 1, 2, \ldots\}$$

for all $y^1, y^2 \in M$.

In this part we shall study the structure of (M, d) from the point of view of DF.

Theorem 1. Let (y^k) be a sequence of elements from M and let

$$y^k \to y \quad \text{in} \quad (M, d) . \tag{1}$$

Let f_k be a WADF of y^k $(k = 1, 2, \dots)$. Then.

(a) y has a WADF f such that $f_k \xrightarrow{w} f$.

(b) If $\bar{f}$ is DF and $f_k \xrightarrow{w} \bar{f}$ then f is a WADF of y.

Proof. By assumption

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n^k) = \int_0^1 h df_k$$

for every continuous function h and for every k. Since h is uniformly continuous on the interval [0, 1] according to (1) for every $\varepsilon > 0$ there exists a k_0 such that for $k \geq k_0$ and for all n we have

$$|h(y_n^k) - h(y_n)| < \varepsilon.$$

Hence for $k \geq k_0$ and each N we obtain

$$\left| \frac{1}{N} \sum_{n=1}^{N} h(y_n^k) - \frac{1}{N} \sum_{n=1}^{N} h(y_n) \right| < \varepsilon. \tag{2}$$

From this for $k \geq k_0$

$$\left| \limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n - \int_0^1 h df_k \right| \leq \varepsilon$$

$$\left| \liminf_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n) - \int_0^1 h df_k \right| \leq \varepsilon$$

so

$$\limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n) - \liminf_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n \leq 2 \cdot \varepsilon.$$

Therefore for every continuous function h there exists a finite limit

$$L(h) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n).$$

Clearly, L is a bounded linear functional on $C[0, 1]$ and $L(h) \geq 0$ for $h \geq 0$, thus by Riesz representation theorem there exists a nondecreasing function $\bar{f}$ on [0, 1] such that

$$L(h) = \int_0^1 h d\bar{f}.$$

If we put $f = \bar{f} - \bar{f}(0)$, then for $h \equiv 1$ we have $f(1) = 1$ and thus by Proposition 1 (e) the function f is a WADF for y. It follows from (2)

$$\lim_{N \to \infty} \int_0^1 hdf_k = \int_0^1 hdf.$$

Therefore

$$f_k \xrightarrow{w} f.$$

If $f_k \xrightarrow{w} \tilde{f}$ and $\tilde{f}$ is a DF, then for every continuity point x of f and of $\tilde{f}$ we have

$$f(x) = \tilde{f}(x).$$

From this part (b) follows.

Theorem 1 has the following immediate cconsequence.

Corollary 1. The set of all $y \in \boldsymbol{M}$, having WADF, is closed in the metric space $(\boldsymbol{M}, d)$.

Let g be a DF. Denote by $\boldsymbol{B}_g$ the set of all sequences $y \in \boldsymbol{M}$, having WADF g.

Theorem 2. The set $\boldsymbol{M}$ is a set of the second Baire category in the space $(\boldsymbol{M}, d)$.

Proof. Let

$$\boldsymbol{M^*} = \{y = (y_n); \quad y_n \in [0, 1]\}.$$

$\boldsymbol{M^*}$ is a complete space, therefore $\boldsymbol{M^*}$ is a space of second Baire category in itself. Let us denote

$$\boldsymbol{H}_k = \{y \in \mathbf{M^*}; \quad y_k = 1\}$$

$$\boldsymbol{H} = \bigcup_{n=1}^{\infty} \boldsymbol{H}_n.$$

Then $\boldsymbol{M^*} = \boldsymbol{M} \cup \boldsymbol{H}$. Let $\boldsymbol{K}(y^0, \delta)$ be a ball in $\boldsymbol{M^*}$. If $y_k^0 < 1$, denote $\eta = \min\{\delta, 1 - y_k^0\}$. For $y \in \boldsymbol{K}(y^0, \eta)$ we have $y_k < 1$, therefore $\boldsymbol{K}(y^0, \eta) \cap \boldsymbol{H}_k = \emptyset$. If $y_k^0 = 1$ denote by $y^{(1)}$ the sequence $(y_n^{(1)})$ where

$$y_n^{(1)} = \begin{matrix} y_n^0 & n \neq k \\ 1 - \dfrac{\delta}{2} & n = k \end{matrix}$$

Then $\boldsymbol{K}\left(y^{(1)}, \dfrac{\delta}{4}\right) \subset \boldsymbol{K}(y^0, \delta)$ and for $y \in \boldsymbol{K}\left(y^{(1)}, \dfrac{\delta}{4}\right)$ we have $y_k < 1$, so $\boldsymbol{K}\left(y^{(1)}, \dfrac{\delta}{4}\right) \cup \cup \boldsymbol{H}_k = \emptyset$. Therefore $\boldsymbol{H}_k$ is a nowhere dense set. From this we get that $\boldsymbol{H}$ is a set of the first Baire category in $\boldsymbol{M^*}$. Therefore $\boldsymbol{M}$ is a set of the second Baire category in itself.

Theorem 3. If g is a continuous DF then $\boldsymbol{B}_g$ is a closed nowhere dense set in that the metric space $(\boldsymbol{M}, d)$.

Proof. We have seen that $\boldsymbol{B}_g$ is a closed set (see Corollary 2). Put

$$\boldsymbol{H}_g = \boldsymbol{M} - \boldsymbol{B}_g .$$

It suffices to prove that $\boldsymbol{H}_g$ is a dense set.
Let $\boldsymbol{K}(y, \varepsilon)$ be an arbitrary ball in $\boldsymbol{M}$. If $y \in \boldsymbol{H}_g$ then $\boldsymbol{H}_g \cap \boldsymbol{K}(y, \varepsilon) \neq \emptyset$. Let $y \notin \boldsymbol{H}_g$. Then $y \in \boldsymbol{B}_g$. Let $\alpha \in (0, 1)$. Denote

$$a = \sup\{\eta \in [0, 1]; \quad g(\eta) = \alpha\} .$$

Then $a > 0$ and from the continuity of g it follows that $g(a) = \alpha$. Thus

$$\alpha < g(a + \delta)$$

for every $\delta > 0$ for which $a + \delta \in [0, 1]$. Let δ satisfy the conditions $0 < \delta < \frac{\varepsilon}{2}$, $a - \delta$, $a + 3\delta \in [0, 1]$.
Define the sequence $y' = (y'_n)$ in such a way that

$$y'_n = \begin{matrix} y_n + 2\delta, & y_n \in [0, a + \delta) \\ y_n , & y_n \notin [0, a + \delta) . \end{matrix}$$

Then $d(y, y') = 2\delta < \varepsilon$, thus $y' \in \boldsymbol{K}(y, \varepsilon)$. But $y'_n < a + \delta \Leftrightarrow y_n < a + \delta \Leftrightarrow y'_n =$ $= y_n + 2\delta$. From this we have

$$\frac{A([0, a + \delta), y', N)}{N} = \frac{1}{N} \sum_{\substack{n \leq N \\ y'_n < a + \delta}} 1 = \frac{1}{N} \sum_{\substack{n \leq N \\ y_n < a - \delta}} 1 \to g(a - \delta) \leq \alpha < g(a + \delta) .$$

From this it follows that g is not a WADF for y'. Therefore $\boldsymbol{H}_g \cap \boldsymbol{K}(y, \varepsilon) \neq \emptyset$, hence $\boldsymbol{H}_g$ is a dense set. From this it follows that $\boldsymbol{B}_g$ is a nowhere dense set.

In the end of this part we shall prove the following theorem about a diagonal sequence.

Theorem 4. Let (y^k) be a Cauchy sequence of elements of $\boldsymbol{M}$. Let f_k bbe a WADF of y^k $(k = 1, 2, \dots)$ and let $f_k \xrightarrow{w} f$, where f is a DF. Then

(a) $$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n^N) = \int_0^1 hdf$$

(b) $$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} h(y_n^n) = \int_0^1 hdf$$

for every continuous function h.

Proof. The function h is uniformly continuous on [0, 1]. Since (y^k) is a Cauchy sequencce for every $\varepsilon > 0$ there exists a k_0 such that for each $k', k'' \geq k_0$ we have

$$|h(y_n^{k'}) - h(y_n^{k''})| < \varepsilon \tag{3}$$

By P. Levy convergence theorem

$$\lim_{k\to\infty} \int_0^1 hdf_k = \int_0^1 hdf.$$

Thus, there exists $k_1 > k_0$ such that

$$\left| \int_0^1 hdf_k - \int_0^1 hdf \right| < \varepsilon \tag{4}$$

for every $k \geq k_1$.
By assumption there exists $N_0 \geq k_1$ such that

$$\left| \frac{1}{N} \sum_{n=1}^{N} h(y_n^{k_1}) - \int_0^1 hdf_{k_1} \right| < \varepsilon \tag{5}$$

for every $N \geq N_0$. By (3), (4), (5) we have

$$\left| \frac{1}{N} \sum_{n=1}^{N} h(y_n^N) - \int_0^1 hdf \right| \leq \left| \frac{1}{N} \sum_{n=1}^{N} h(y_n^N) - \frac{1}{N} \sum_{n=1}^{N} h(y_n^{k_1}) \right| +$$

$$+ \left| \frac{1}{N} \sum_{n=1}^{N} h(y_n^{k_1}) + \int_0^1 hdf_{k_1} \right| + \left| \int_0^1 hdf_{k_1} - \int_0^1 hdf \right| \leq 3 \cdot \varepsilon$$

Thus part (a) is proved. The function h is bounded on $[0, 1]$. From this, according to (a), part (b) follows. Thus the proof is finished.

3. Transformations of sequences and ADF preserving Function

Let $y \in M$ and f: $[0, 1] \to [0, 1]$. Denote $f(y) = (f(y_n))$. In this part we examine the properties of the sequence $f(y)$ if y has WADF or ADF.

Directly from the Riesz representation theorem we get:

Theorem 5. Let f: $[0, 1] \to [0, 1]$ be a continuous function and $y \in M$ have WADF. Then $f(y)$ has WADF.

Theorem 6. Let y have continuous ADF g and f: $[0, 1] \to [0, 1]$ be a continuous function. Then the sequence $f(y)$ has a continuous ADF if and only if

$$\lambda_g^2(\{(x, y);\ \ f(y) - f(x) \in Z \wedge x, y \in [0, 1]\}) = 0 \tag{6}$$

where λ_g^2 is a 2-dimensional Lebesgue—Stielties measure.

Proof. By the Wiener—Schoenberg theorem (see [1] p. 55) there exists a finite limit

$$w_m = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} e^{2\pi i m f(y_n)} = \int_0^1 e^{2\pi i m f(x)}\, dg(x)$$

for every m. By Wiener—Schoenberg theorem $f(y)$ has a continuous ADF if and only if

$$\lim_{M\to\infty} \frac{1}{M}\sum_{m=1}^{M} |w_m|^2 = 0. \tag{7}$$

By theorem on dominated convergence we have

$$\lim_{M\to\infty} \frac{1}{M}\sum_{m=1}^{M} |w_m|^2 = \lim_{M\to\infty} \frac{1}{M}\sum_{m=1}^{M} w_m \cdot \bar{w}_m =$$

$$= \lim_{M\to\infty} \frac{1}{M}\sum_{m=1}^{M} \int_0^1\int_0^1 e^{2\pi i m(f(x)-f(y))}\, dg(x)dg(y) =$$

$$= \int_0^1\int_0^1 \left(\lim_{M\to\infty} \frac{1}{M}\sum_{m=1}^{M} e^{2\pi i m(f(x)-f(y))}\right) dg(x)dg(y) =$$

$$= \iint_{\{(x,y);\, f(x)-f(y)\in Z\}} 1\, dg(x)dg(y) = \lambda_g^2(\{(x,y);\quad f(x)-f(y)\in Z\}).$$

Thus by (7) the proof is finished.

Definition 2. Let g be a DF. The function f: $[0, 1] \to [0, 1]$ is said to be a g preserving function if the following assertion holds:

If g is an ADF of y, then g is also ADF of $f(y)$ for every $y \in \boldsymbol{M}$.

If $g(x) = x$, f is said to be a u. d. preserving function. Analogously as in the paper [3] we can prove the following:

Proposition 2. A continuous function f: $[0, 1] \to [0, 1]$ is g-preserving, where g is a continuous DF if and only if

(a) $$\int_0^1 h \circ f dg = \int_0^1 h dg$$

for every $h \in C[0, 1]$.

(b) $$\lambda_g(f^{-1}(I)) = \lambda_g(I)$$

for every interval $I \subset [0, 1]$.

Corollary 3. A continuous function f is g-preserving if and only if there exists a $y \in \boldsymbol{M}$, having ADF g, such that $f(y)$ has ADF g.

Corollary 4. If for a continuous function f: $[0, 1] \to [0, 1]$ there exists $x \in [0, 1]$ such that the sequence $x, f(x), f(f(x)), \ldots,$ has a continuous ADF, then f is g-preserving.

Let g be a continuous DF. Then g is a surjection and we can define the function

$$\bar{g}(x) = \sup\{\alpha \in [0, 1]; \quad g(\alpha) = x\}$$

From the continuity g we get $g(\bar{g}(x)) = x$.

Lemma 1. For x_1, $x_2 \in [0, 1]$ we have

$$g(x_1) \leq x_2 \Leftrightarrow x_1 \leq \bar{g}(x_2)$$

The proof is trivial.

Proposition 3.

(a) If y has ADF g, then $g(y)$ is a uniformly distributed sequence.

(b) If y is a uniformly distributed sequence, then g is and ADF for $\bar{g}(y)$.

Proposition 3 is an immediate consequence of Lemma 1.

Then, if f is u. d. preserving and y has ADF g, $g(y)$ is uniformly distributed sequence, $f \circ g(y)$ is a uniformly distributed sequence. Therefore $\bar{g} \circ f \circ g(y)$ has ADF g. Therefore $\bar{g} \circ f \circ g$ is a g-preserving function. We can prove also that if f_1 is g-preserving, the $g \circ f_1 \circ g$ is u. d. preserving. Thus we have proved:

Corollary 5. If g is continuous, increasing DF, then every g-preserving function can be written in the form $g^{-1} \circ f \circ g$, where f is u. d. preserving.

Using the results from [3] we obtain

Theorem 7. If f is a continuous, monotone g-preserving function, then

$$f(x) = x \quad \text{or} \quad f(x) = g^{-1}(1 - g(x)).$$

REFERENCES

1. Kuipers Z.—Niederreiter H.: Uniform distribution of sequences. John Wiley et Sons. New York — London — Sydney — Toronto 1974.
2. Hlawka E.: Theorie der Gleichverteilung. Bibliographisches Institut Manheim — Wien — Zurich 1979.
3. Porubský, Š.—Šalát, T.—Strauch, O.: Functions that preserve uniform distribution. To appear.

Author s addres:
Milan Paštéka
Katedra algebry a teórie čísel MFF-UK
Matematický pavilón
Mlynská dolina
Bratislava
842 15

Reccived: 18. 11. 1985

SÚHRN

DISTRIBUČNÉ FUNKCIE POSTUPNOSTÍ

Milan Paštéka, Bratislava

V tejto práci zavádzame okrem iného pojem slabej asymptotickej distribučnej funkcie pre postupnosti prvkov intervalu [0, 1]. Vyšetrujeme topologické vlastnosti asymptotických distribučných a slabých asymptotických distribučných funkcií. V práci je okrem iného dokázané, že množina všetkých postupností intervalu, ktoré majú slabú asymptotickú distribučnú funkciu, je uzavretá vzhľadom na suprémovú metriku, ďalej, že množina všetkých postupností, ktoré majú spojitú asymptotickú funkciu g, je tiež uzavretá, vzhľadom na suprémovú metriku. V poslednej časti sa zaoberáme transformáciami postupností. Mimo iného je dokázaná veta o jednoznačnom rozklade funkcií, ktoré zachovávajú asymptotické distribučné funkcie.

РЕЗЮМЕ

ФУНКЦИИ РАСПРЕДЕЛЕНИЙ ПОСЛЕДОВАТЕЛЬНОСТЕЙ

Милан Паштека, Братислава

В настоящей работе используется понятие слабой аимптотической функции распределения последовательности элементов интервала [0, 1]. Исследуются топологические свойства асимптотических функций распределения. В работе кроме того доказывается, что множество всех последовательностей интервала, имеющих слабую асимптотическую функцию распределения, замкнуто в сооответствующей метрике, и далее, что множество всех последовательностей, имеющих непрерывную асимптотическую функцию тоже замкнуто. В последней части изучаются преобразования последовательностей и доказывается теорема об однозначном разбиении функций, которые сохраняют асимптотические функции распределения.