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ON THE KOLMOGOROV CONSISTENCY THEOREM FOR RIESZ SPACE VALUED MEASURES

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Let I be an index set such that for every finite non-empty $\alpha \subset I$, $\alpha = \{i_1, \ldots, i_n\}$ there is a Borel probability measure μ_α: $B_n \to R$. If the measures $(\mu_\alpha)_{\alpha \subset I}$ form a consistent system of measures, then the classical Kolmogorov theorem states that on the space R^I there is a measure μ such that $\mu(\pi_\alpha^{-1}(E)) = \mu_\alpha(E)$ for every finite $\alpha \subset I$, $E \in B$. In the paper we prove a generalization of the theorem in the case that μ_α: $B_n \to X$, where X is a Riesz space (i.e. a linear lattice) of some type.

1. Compact approximation

1.1 Definition. A linear space X will be called a Riesz space if

a) X is a lattice

b) for any x, $y \in X$ such that $x \leq y$, any $z \in X$ and any $c \in R$, $c \geq 0$ it is $x + z \leq y + z$, $c \cdot x \leq c \cdot y$.

1.2 Definition. Let M be a set, $A \subset 2^M$ be an algebra and X be a Riesz space. A function μ: $A \to X$ will be called an X-valued content, if

(i) $\mu(\emptyset) = 0$

(ii) For any set $E \in A$ it is $\mu(E) \geqq 0$.

(iii) For any sets $E_i \in A$ $(i = 1, 2, \ldots)$ such that $E_n \cap E_m = \emptyset$ $(n \neq m)$ and for any positive integer k it holds

$$\mu\left(\bigcup_{i=1}^{k} E_i\right) = \sum_{i=1}^{k} \mu(E_i) \qquad \text{(additivity)}$$

1.3 Definition. Let M be a set, $A \subset 2^M$ be an algebra and X be a Riesz space. An X-valued content μ: $A \to X$ will be called an X-valued measure, if

(iv) for arbitrary sets $F_i \in A$ $(i = 1, 2, \ldots)$ such that $E_n \cap E_m = \emptyset$ $(n \neq m)$ it holds

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right) = \bigvee_{n=1}^{\infty} \sum_{i=1}^{n} \mu(E_i) \qquad (\sigma\text{-additivity})$$

μ will be called a continuous X-valued content, if

(v) for arbitrary sets $E_i \in A$ $(i = 1, 2, \ldots)$ such that $E_{i+1} \subset E_i$ $(i = 1, 2, \ldots)$ and $\bigcap_{i=1}^{\infty} E_i = \emptyset$, it is

$$\bigwedge_{i=1}^{\infty} \mu(E_i) = 0 \qquad \text{(continuity)}$$

1.4 Remark. A Riesz space X will be called σ-complete, if any non-empty, countable, bounded set has the supremum and the infimum. X is weakly σ-distributive, if it is true:

$$(a_{ij})_{ij} \text{ bounded, } a_{ij} \searrow 0 \ (j \to \infty,\ i = 1, 2, \ldots) \Rightarrow$$

$$\Rightarrow \bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} a_{i,\varphi(i)} = 0$$

It is easy to prove the following lemma:

Lemma A. Let X be a σ-complete, weakly σ-distributive Riesz space. Let $\{a_{nij}\}_{n,i,j=1}^{\infty}$ be a bounded sequence such that $a_{nij} \searrow 0$ $(j \to \infty,\ i = 1, 2, \ldots, n = 1, 2, \ldots)$. Then to any $a \in X$, $a > 0$ there exists a bounded sequence $\{a_{ij}\}_{ij=1}^{\infty}$ $a_{ij} \searrow 0$ $(j \to \infty,\ i = 1, 2, \ldots)$ such that for any $\varphi\colon N \to N$ it is

$$a \wedge \sum_{n=1}^{\infty} \bigvee_{i=1}^{\infty} a_{ni\varphi(i+n)} \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

(Proof: See [4] Proposition 3.)

1.5 Lemma. Let M be a set, $A \subset 2^M$ be an algebra, X be a σ-complete Riesz space and $\mu\colon A \to X$ be an X-valued content. Then μ is an X-valued measure if and only if it is a continuous X-valued content.

Proof. $(\Rightarrow)$

Let $\{E_i\}_{i=1}^{\infty}$ be a sequence such that $E_{i+1} \subset E_i$, $E_i \in A$ $(i = 1, 2, \ldots)$ and $\bigcap_{i=1}^{\infty} E_i = \emptyset$. Put $F_i = E_i - E_{i+1}$ $(i = 1, 2, \ldots)$. Evidently $F_n \cap F_m = \emptyset$ $(n \neq m)$, hence

$$\mu\left(\bigcup_{i=1}^{\infty} F_i\right) = \bigvee_{n=1}^{\infty} \sum_{i=1}^{n} \mu(F_i)$$

Further $E_1 = \bigcup_{n=1}^{\infty} F_n$ and

$$0 = \mu\left(E_1 - \bigcup_{i=1}^{\infty} F_i\right) = \mu(E_1) - \mu\left(\bigcup_{i=1}^{\infty} F_i\right) =$$

$$= \mu(E_1) - \bigvee_{n=1}^{\infty} \sum_{i=1}^{n} \mu(F_i) = \bigwedge_{n=1}^{\infty} \left(\mu(E_i) - \sum_{i=1}^{n} \mu(F_i)\right) =$$

$$= \bigwedge_{n=1}^{\infty} (\mu(E_1) - (\mu(E_1 \backslash E_n))) = \bigwedge_{n=1}^{\infty} \mu(E_n)$$

($\Leftarrow$)

Let $\{E_i\}_{i=1}^{\infty}$ be a sequence such that $E_i \in A$ $(i = 1, 2, \ldots)$, $E_n \cap E_m = \emptyset$ $(n \neq m)$ and $E = \bigcup_{i=1}^{\infty} E_i \in A$. Denote

$$F_n = E - \bigcup_{i=1}^{n} E_i \qquad (n = 1, 2, \ldots).$$

Then $F_n \in A$, $F_{n+1} \subset F_n$ $(n = 1, 2, \ldots)$ and $\bigcap_{n=1}^{\infty} F_n = \emptyset$, hence

$$0 = \bigwedge_{n=1}^{\infty} \mu(F_n) = \bigwedge_{n=1}^{\infty} \mu\left(E - \bigcup_{i=1}^{n} E_i\right) = \bigwedge_{n=1}^{\infty} \left(\mu(E) - \mu\left(\bigcup_{i=1}^{n} E_i\right)\right) =$$

$$= \mu(E) - \bigvee_{n=1}^{\infty} \left(\sum_{i=1}^{n} \mu(E_i)\right)$$

Hence

$$\mu(E) = \mu\left(\bigcup_{i=1}^{\infty} E_i\right) = \bigvee_{n=1}^{\infty} \left(\sum_{i=1}^{n} \mu(E_i)\right).$$

Q.E.D.

1.6 Definition. Let M be a set $\mathscr{A} \subset 2^M$ be an algebra, X be a Riesz space and μ: $A \to X$ be an X-valued content. A system $\mathscr{C} \subset \mathscr{A}$ approximates a set $B \in \mathscr{A}$, if there exists a bounded sequence $\{a_{ij}\}_{i,j=1}^{\infty}$ such that $a_{ij} \searrow 0$ $(j \to \infty, i = 1, 2, \ldots)$ and such that to any φ: $N \to N$ there exists $C \in \mathscr{C}$, $C \subset B$ and

$$\mu(B - C) \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

The system $\mathscr{C}$ approximates a system $\mathscr{B} \subset \mathscr{A}$, if it approximates each set $B \in \mathscr{B}$.

1.7 Definition. Let M be a set, $\mathscr{A} \subset 2^M$ be an algebra, X be a Riesz space and μ: $\mathscr{A} \to X$ be an X-valued content. A system $\mathscr{C} \subset \mathscr{A}$ is compact, if for every sequence $\{C_i\}_{i=1}^{\infty}$, $C_i \in \mathscr{C}$ $(i = 1, 2, \ldots)$ such that $\bigcap_{i=1}^{n} C_i \neq \emptyset$ $(n = 1, 2, \ldots)$ it is $\bigcap_{i=1}^{\infty} C_i \neq \emptyset$. A system $\mathscr{B} \subset \mathscr{A}$ is compactly approximable, if there exists a compact system $\mathscr{C} \subset \mathscr{A}$ which approximates $\mathscr{B}$.

1.8 Theorem (Alexandrov). Let M be a set, $\mathscr{A} \subset 2^M$ be an algebra, X be a σ-complete, weakly σ-distributive Riesz space and μ: $\mathscr{A} \to X$ be an X-valued content. Let $\mathscr{A}$ be compactly approximable. Then μ is continuous and hence μ is an X-valued measure.

Proof. Let $E_i \in \mathscr{A}$, $E_{i+1} \subset E_i$ $(i = 1, 2, \ldots)$ and $\bigcap_{i=1}^{\infty} E_i = \emptyset$. Since $\mathscr{A}$ is a compactly approximable algebra, there exists a compact system $\mathscr{C} \subset \mathscr{A}$ such that to any set F_n $(n = 1, 2, \ldots)$ there exists $C_n \subset \mathscr{C}$ such that $C_n \subset E_n$ and $\mu(E_n - C_n) \leqq \bigvee_{n=1}^{\infty} a_{ni\varphi(i)}$.

Since $\bigcap_{n=1}^{\infty} E_n = \emptyset$, then $\bigcap_{n=1}^{\infty} C_n = \emptyset$ and it follows by the compactness of $\mathscr{C}$ that there exists $m \in Z^+$ such that $\bigcap_{i=1}^{m} C_i = \emptyset$. Evidently $\bigcap_{i=1}^{n} C_i = \emptyset$ for any $n > m$. Hence for any $n > m$ it is

$$\bigcap_{k=1}^{n} E_k = \bigcap_{k=1}^{n} E_k - \bigcap_{k=1}^{n} C_k \subset \bigcup_{k=1}^{n} (E_k - C_k).$$

It follows that

$$\mu(E_n) = \mu\left(\bigcup_{k=1}^{n} (E_k - C_k)\right) \leqq \sum_{k=1}^{n} \bigvee_{i=1}^{\infty} a_{ki\varphi(i+n)}.$$

By Lemma A to the element $a = \mu(X)$ there exists a sequence $\{a_{ij}\}_{i,j=1}^{\infty}$ which is bounded and such that $a_{ij} \searrow 0$ $(j \to \infty,\ i = 1, 2, \ldots)$. Hence

$$\mu(E_n) = \mu(X) \wedge \sum_{k=1}^{n} \bigvee_{i=1}^{\infty} a_{ki\varphi(i+n)} \leqq \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for any $\varphi\colon N \to N$. Therefore

$$\bigwedge_{n=1}^{\infty} \mu(E_n) \leqq \bigwedge_{\varphi \in N^N} \bigvee_{i=1}^{\infty} a_{i\varphi(i)} = 0$$

By Lemma 1.5 μ is an X-valued measure. Q.E.D.

1.9 Lemma. Let M be a set, $\mathscr{A} \subset 2^M$ be a σ-algebra, X be a σ-complete, weakly σ-distributive Riesz space and $\mu\colon \mathscr{A} \to X$ be an X-valued content. Let $\mathscr{S} \subset \mathscr{A}$ approximate $\mathscr{B} \subset \mathscr{A}$. Then $\mathscr{S}^\sigma$ approximates $\mathscr{B}^\sigma$ and $\mathscr{S}^\delta$ approximates $\mathscr{B}^\delta$. ($\mathscr{B}^\sigma$ is the system generated by $\mathscr{B}$ and closed under countable unions and $\mathscr{B}^\delta$ is the generated system closed under countable intersections.)

Proof. 1° (unions). If $B \in \mathscr{B}^\sigma$, then there exist $B_k \in \mathscr{B}$ $(k = 1, 2, \ldots)$ such that $B = \bigcup_{k=1}^{\infty} B_k$. To any positive integer k there exists $\{a_{kij}\}_{i,j=1}^{\infty}$ which is bounded, $a_{kij} \searrow 0$ $(j \to \infty,\ i = 1, 2, \ldots)$ and such that the following is true: There exist $S_k \in \mathscr{S}$ such that $S_k \subset B_k$ and

$$\mu(B_k - S_k) = \bigvee_{i=1}^{\infty} a_{ki\varphi(k+i)}.$$

By Lemma A (see 1.4) there exists a bounded sequence $\{a_{ij}\}_{i,j=1}^{\infty}$ such that

$$\sum_k \bigvee_i a_{ki\varphi(k+i)} = \bigvee_i a_{i\varphi(i)}.$$

Further, $S = \bigcup_{k=1}^{\infty} S_k \subset \bigcup_{k=1}^{\infty} B_k = B$ and to any $\varphi: N \to N$

$$\mu(B - S) \leqq \mu\left(\bigcup_{k=1}^{\infty} (B_k - S_k)\right) \leqq \mu(M) \wedge \sum_{k=1}^{\infty} \bigvee_{i=1}^{\infty} a_{ki\varphi(i+k)} \leqq \bigcup_{i=1}^{\infty} a_{i\varphi(i)}$$

2° (intersections). If $B \in \mathscr{B}^{\delta}$, then there exist $B_k \in \mathscr{B}$ $(k = 1, 2, \ldots)$ such that $B = \bigcap_{k=1}^{\infty} B_k$. To any positive integer k there exists $\{a_{kij}\}_{i,j=1}^{\infty}$, which is bounded, $a_{kij} \searrow 0$ $(j \to \infty,\ i,\ k = 1, 2, \ldots)$ and there is $S_k \in \mathscr{S}$, $S_k \subset B_k$ that for every $\varphi: N \to N$

$$\mu(B_k - S_k) \leqq \bigvee_{i=1}^{\infty} a_{ki\varphi(i+k)}$$

Hence by Lemma A there exists a bounded sequence $\{a_{ij}\}_{i,j=1}^{\infty}$ such that $a_{ij} \searrow 0$ $(j \to \infty,\ i = 1, 2, \ldots)$ and

$$\mu(M) \wedge \sum_i \bigvee_k a_{ki\varphi(k+i)} \leqq \bigvee_i a_{i\varphi(i)}$$

Then $S = \bigcap_{k=1}^{\infty} S_k \subset \bigcap_{k=1}^{\infty} B_k = B$ and

$$\mu(B - S) \leqq \mu\left(\bigcup_{k=1}^{\infty} (B_k - S_k)\right) = \mu(M) \wedge \sum_{k=1}^{\infty} \bigvee_{i=1}^{\infty} a_{ki\varphi(i+k)} = \bigvee_{i=1}^{\infty} a_{i\varphi(i)}$$

for any $\varphi: N \to N$. Q.E.D.

1.10 Remark. If we assume in Lemma 1.9 that $\mathscr{A}$ is an algebra, then we can prove that $\mathscr{S}^{\cap}$ (the least system over $\mathscr{S}$ closed under finite intersections) approximates $\mathscr{B}^{\cap}$ and $\mathscr{S}^{\cup}$ (the least system over $\mathscr{S}$ closed under finite unions) approximates $\mathscr{B}^{\cup}$.

1.11 Lemma. *Let M be a set, $\mathscr{A} \subset 2^M$ be an algebra, X be a Riesz space and $\mu: \mathscr{A} \to X$ be an X-valued content. Let I be an index set. Let $\mathscr{S}_a \subset \mathscr{A}$ approximate $\mathscr{B}_a \subset \mathscr{A}$ for any $a \in I$. Then $\bigcup_{a \in I} \mathscr{S}_a$ approximates $\bigcup_{a \in I} \mathscr{B}_a$.*

Proof. To any $B_0 \in \bigcup_{a \in I} \mathscr{B}_a$ there exists $a_0 \in I$ such that $B_0 \in \mathscr{B}_{a_0}$. Since $\mathscr{S}_{a_0}$ approximates $\mathscr{B}_{a_0}$, $\bigcup_{a \in I} \mathscr{S}_a$ approximates $\mathscr{B}_0$.

1.12 Lemma. Let M be a set, $\mathscr{A} \subset 2^M$ be an algebra, X be a σ-complete, weakly σ-distributive Riesz space, $\mu: A \to X$ be an X-valued content. Let I be an index set. Let $\mathscr{S}_a \subset \mathscr{A}$ approximate $\mathscr{B}_a \subset \mathscr{A}$ for any $a \in I$. Then $\left(\bigcup_{a\in I} \mathscr{S}_a\right)^{\cap\cup}$ approximates the algebra $\mathscr{A}\left(\bigcup_{a\in I} \mathscr{B}_a\right)$ generated by $\bigcup_{a\in I} \mathscr{B}_a$.

Proof. We shall use the next lemma (see [1], Corollary (0.2)): Let $\{\mathscr{A}_a;\ a \in I\}$ be a system of subalgebras of an algebra $\mathscr{A}$. Then

$$\mathscr{A}\left(\bigcup_{a\in I} \mathscr{A}_a\right) = \bigcup_{I_0 \in J_0} \left(\bigcup_{a\in I_0} \mathscr{A}_a\right)^{\cap\cup},$$

where $J_0 = \{I_0 \subset I;\ I_0 \text{ is finite}\}$.

By Lemma 1.11 the system $\bigcup_{a\in I} \mathscr{S}_a$ approximates $\bigcup_{a\in I} \mathscr{B}_a$. By Lemma 1.9 the system $\left(\bigcup_{a\in I} \mathscr{S}_a\right)^{\cap\cup}$ approximates $\left(\bigcup_{a\in I} \mathscr{B}_a\right)^{\cap\cup}$, hence also

$$\mathscr{A}\left(\bigcup_{a\in I} \mathscr{B}_a\right) = \bigcup_{I_0 \in J_0} \left(\bigcup_{a\in I_0} \mathscr{B}_a\right)^{\cap\cup}.$$

Q.E.D.

1.13 Corollary. Let M be a set, $\mathscr{A} \subset 2^M$ be an algebra, X be a σ-complete, σ-distributive Riesz space and $\mu: \mathscr{A} \to X$ be an X-valued content. Let $\{\mathscr{B}_a;\ a \in I\}$ be a system of algebraically σ-independent algebras ($\mathscr{B}_a \subset \mathscr{A}$) and $\mathscr{B}_a$ be a compactly approximable algebra for any $a \in I$. Then the algebra $\mathscr{A}\left(\bigcup_{a\in I} \mathscr{B}_a\right)$ is compactly approximable.

Proof. Let $\mathscr{C}_a$ be a compact system approximating $\mathscr{B}_a$ for any a.

I. Then by Lemma 1.12 $\left(\bigcup_{a\in I} \mathscr{C}_a\right)^{\cap\cup}$ approximates the algebra $\mathscr{A}\left(\bigcup_{a\in I} \mathscr{B}_a\right)$. By [1] Lemma (1.3), Lemma 1.4 and Theorem 1.6, $\left(\bigcup_{a\in I} \mathscr{C}_a\right)^{\cap\cup}$ is a compact system.

Q.E.D.

1.14 Theorem. Let $\{\mathscr{A}_a;\ a \in I\}$ be a system of algebraically σ-independent algebras, X be a σ-complete, weakly σ-distributive Riesz space and

$$\mu: \mathscr{A}\left(\bigcup_{a\in I} \mathscr{A}_a\right) \to X$$

be an X-valued content. Let $\mathscr{A}_a$ be compactly approximable for any $a \in I$. Then $\mathscr{A}\left(\bigcup_{a\in I} \mathscr{A}_a\right)$ is compactly approximable and μ is an X-valued measure.

Proof. By Corollary 1.13 the algebra $\mathscr{A}\left(\bigcup_{\alpha\in I}\mathscr{A}_\alpha\right)$ is compactly approximable and by the Alexandrov theorem 1.8 μ is an X-valued measure.

Q.E.D.

2. Kolmogorov theorem

Let I be a directed set (i.e. such a partially ordered set that for every $\alpha, \beta\in I$ there exists $\gamma\in I$ with $\alpha \leqq \gamma$, $\beta \leqq \gamma$). By a projective system we mean a family $\{X_\alpha;\ \alpha\in I\}$ of sets with a family of mappings $\pi_{\beta,\alpha}: X_\beta \to X_\alpha$ $(\alpha \leqq \beta)$ such that $\pi_{\alpha,\alpha}$ is the identity map for every $\alpha\in I$ and $\pi_{\beta,\alpha}\circ\pi_{\gamma,\beta} = \pi_{\gamma,\alpha}$ whenever $\alpha, \beta, \gamma\in I$, $\alpha \leqq \beta \leqq \gamma$. The projective limit of a projective system $\{X_\alpha;\ \alpha\in I\}$ is the set $X_\infty = \left\{x\in \mathop{\times}_{\alpha\in I} X_\alpha;\ \pi_{\beta,\alpha}(x_\beta) = x_\alpha \text{ for any } \alpha, \beta\in I \text{ such that } \alpha \leqq \beta\right\}$.

In this section $\{X_\alpha;\ \alpha\in I\}$ will be a projective system of compact topological spaces, $\mathscr{B}(X_\alpha)$ will mean the family of Baire subsets of X_α and $\pi_\alpha: X_\infty \to X_\alpha$ will be the projection.

2.1 Definition. Let $\{X_\alpha;\ \alpha\in I\}$ be a projective system of spaces. Let $\mu_\alpha: \mathscr{B}(X_\alpha) \to X$ be an X-valued content for any $\alpha\in I$. $\{\mu_\alpha;\ \alpha\in I\}$ is called a consistent system of contents, if for any $\alpha_1 < \alpha_2\in I$ and for any set $E\in\mathscr{B}(X_\alpha)$ it is

$$\mu_{\alpha_2}(\pi_{\alpha_2\alpha_1}^{-1}(E)) = \mu_{\alpha_1}(E)$$

2.2 Definition. Let X_∞ be a projective limit of $\{X_\alpha;\ \alpha\in I\}$. Let $M = \{\mu_\alpha;\ \alpha\in I\}$ be a consistent system of measures with values in a Riesz space X $(\mu_\alpha: \mathscr{B}(X_\alpha) \to X)$. Define and X-valued content $\mu: X_\infty \to X$ induced by the system M in the following way: $E = \pi_\alpha^{-1}(F)$, $F\in\mathscr{B}_\alpha$, $\alpha\backslash\backslash$ we define $\mu(E) = \mu_\alpha(\pi_\alpha^{-1}(F))$.

2.3 Lemma. $\mathscr{B}(X_\alpha)$ is a compactly approximable system (See [3], Theorem 2.)

2.4 Lemma. Let $S = \{X_\alpha;\ \alpha\in I\}$ be a projective system, X a be Riesz space, $(\mu_\alpha)_{\alpha\in I}$ be a consistent system of X-valued measures and μ be the induced X-valued content. Then $\pi_\alpha^{-1}(\mathscr{B}(X_\alpha))$ is a compactly approximable system for any $\alpha\in I$ and $\pi_\alpha^{-1}(\mathscr{B}(X_\alpha))$ is an algebra.

Proof. By Lemma 2.3 and Definition 2.2 $\mathscr{B}(X_\alpha)$ is a compactly approximable system. Evidently $\pi_\alpha^{-1}(\mathscr{B}(X_\alpha))$ is a compactly approximable system for any $\alpha\in I$. Q.E.D.

2.5 Theorem (A generalized Kolmogorov theorem for measures with values in Riesz spaces). Let X be a σ-complete, weakly σ-distributive Riesz space, $(I, <)$ be a directed set, $S = \{X_\alpha;\ \alpha\in I\}$ be a projective system. Let X_∞ be a projective limit of this system, $M = \{\mu_\alpha;\ \mathscr{B}(X_\alpha) \to X;\ \alpha\in I\}$ be a consistent system of X-valued contents, $\mu: \mathscr{A}(X_\infty) \to X$ be the induced X-valued content. Then μ is an X-valued measure.

Proof. By 2.3 and 2.4 $\{\pi_\alpha^{-1}(\mathscr{B}(X_\alpha));\ \alpha \in I\}$ is a system of compactly approximable algebras. $\pi_\alpha^{-1}(\mathscr{B}(X_\alpha))$ are algebraically σ-independent algebras. By 1.14 $\mathscr{A}(X_\alpha)$ is compactly approximable and so μ is an X-valued measure.

Q.E.D.

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РЕЗЮМЕ

О ТЕОРЕМЕ КОЛМОГОРОВА О СОГЛАСОВАНИИ ДЛЯ МЕРЫ С ЗНАЧЕНИЯМИ В ПРОСТРАНСТВЕ РИСА

Юрай Риечан, Братислава

В работе доказывается теорема Колмогорова для проективных систем мер с значениями в σ-полном, слабо σ-дистрибутивном пространстве Риса.

SÚHRN

O KOLMOGOROVOVEJ VETE O KONZISTENCII PRE MIERY S HODNOTAMI V RIESZOVOM PRIESTORE

Juraj Riečan, Bratislava

V práci sa dokazuje Kolmogorovova veta pre projektívne systémy mier s hodnotami v σ-úplnom, slabo σ-distributívnom Rieszovom priestore.