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ON THE THEOREM OF POINCARÉ FOR STOCHASTIC TRANSFORMATIONS (MARKOV CHAINS)

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The notion of transformation from X to Y may be generalized in the following way: instead of assigning a "definite" element y from Y to each element x from X, we assign an "indefinite" one, namely a probability measure on Y. We call such a transformation stochastic; an exact definition will be given below. An equivalent concept had already been introduced some time ago, in connection with the theory of Markov chains with the general state space (cf. [1], [3], [4], papers [7], [8]). Our approach is slightly different in motivation; it deals with the problems of ergodic theory and was for the first time used in the thesis [5] of P. Maličký. We use his definition of the stochastic transformation, which was inspired by the notion of polymorphism of Lebesgue space (in the sense of Rohlin [6]) introduced in the A. N. Vershik's paper [9]. Similar to our concept is also G. Choquet's one of diffusion [2]. In this paper we study some recurrence properties of stochastic transformations, namely the recurrence theorem of Poincaré.

Let E be a set; E^c denotes a complement of E in some suitable space X. By χ_E we understand an indicator (characteristic function) of E. The set of all nonnegative integers is denoted by N; real interval $\langle 0, 1\rangle$ by I. A measurable space is a couple $(X, \mathscr{S})$, where X is a set and $\mathscr{S}$ is a σ-algebra of subsets of X. A probability space is a triple $(X, \mathscr{S}, m)$, where $(X, \mathscr{S})$ is a measurable space and m is a probability measure defined on $\mathscr{S}$. We shall frequently omit the word "probability" in the expression "probability measure"; almost all measures considered are probability measures.

The symbol $\int_A f(x)m\,(\mathrm{d}x)$ is used in the usual sense; when $A = X$, it reduces to $\int f(x)m\,(\mathrm{d}x)$ or $m(f)$, the latter in the Bourbaki fashion. By ε_x we denote a Dirac measure, concentrated with unit mass in the point x from X. A symbol $(X^N, \mathscr{S}^N)$ will denote a measurable space consisting of all sequences with the elements from A; σ-algebra $\mathscr{S}^N$ is generated by $\mathscr{S}$-measurable cylinders.

Considering the set X and two measures m_1, m_2 defined on it, in general the

σ-algebras $\mathscr{S}_1$, $\mathscr{S}_2$ — the domains of m_1, m_2 — may be distinct: but we suppose that all measures on X are defined on the same (and usually rich enough) σ-algebra $\mathscr{S}$. Under this assumption we introduce a symbol $M^1(X, \mathscr{S})$, abbreviated to $M^1(X)$, for the set of all (probability) measures on X (with the domain $\mathscr{S}$).

The set of all measurable functions from $(X, \mathscr{S})$ to I is denoted by $M(X, \mathscr{S})$ or shorter by $M(X)$.

Let $(X, \mathscr{S})$ and $(Y, \mathscr{T})$ be measurable spaces. By a stochastic transformation (mapping) from X to Y (denoted $\Phi: X \Rightarrow Y$) we call a diagram

$$\Phi: (X, \mathscr{S}) \overset{F}{\rightarrow} (Y, \mathscr{T}),$$

where F is an ordinary mapping from X to $M^1(Y)$. F is called the associated mapping with the stochastic transformation Φ. The value of the mapping F in the point x is denoted Fx. If $f \in M(Y)$, then we introduce a notation $\Phi^{-1}f$ for the function from $M(X)$, such that $(\Phi^{-1}f)(x) = Fx(f)$. Specially, when $E \in \mathscr{T}$, then we define a function $\Phi^{-1}E$ from $M(X)$ by the relation $\Phi^{-1}E = \Phi^{-1}\chi_E$.

A stochastic transformation Φ is called measurable (according to $(X, \mathscr{S})$, $(Y, \mathscr{T})$), iff for every $f \in M(Y)$ is $\Phi^{-1}f \in M(X)$. Clearly, stochastic transformation Φ is measurable, iff for every $E \in \mathscr{T}$ is $\Phi^{-1}E$ from $M(X)$, because every measurable function is the limit of linear combinations of characteristic functions of measurable sets, and the integral is linear.

Examples. If $X = Y = \{1, \ldots, n\}$, then stochastic transformation Φ is expressed by the transition matrix of the homogenous Markov chain with the finite state space (cf. [1]).

If $f: X \rightarrow Y$ is an ordinary mapping, then we define stochastic transformation $\Phi_f: X \Rightarrow Y$ by defining the value of its associated mapping F_f to be $F_f x = \varepsilon_{f(x)}$. In this way we obtain a natural insertion of the set of all mappings from X to Y to the set of all stochastic transformations from X to Y. This insertion has functorial properties in the sense of the corresponding categories; moreover, all properties defined below are well-defined in the sense that this insertion preserves the analogical property for the ordinary transformations (e.g. when the mapping f is ergodic in the usual sense, then Φ_f is ergodic according to our definition of ergodicity of stochastic transformations), or are at least "almost-well-defined", i.e. the property is preserved with the probability 1.

Namely the notion of measurable stochastic transformation has been already introduced under different names in the theory of Markov chains with a general state space ([1], [4]).

Let $(X_1, \mathscr{S}_1)$, $(X_2, \mathscr{S}_2)$, $(X_3, \mathscr{S}_3)$ be measurable spaces, let $\Phi_1: X_1 \Rightarrow X_2$, $\Phi_2: X_2 \Rightarrow X_3$ be stochastic transformations. Then a composition Φ of stochastic transformations Φ_1, Φ_2 (denoted by $\Phi = \Phi_2\Phi_1$) is a stochastic transformation with an associated mapping F, defined by

$$Fx(E)=\int_{X_2} F_2y(E)F_1y(\mathrm{d}y) \text{ for all } E\in\mathcal{S}_3\,.$$

The correctness of the definition and the associativity of the operation of composition is obvious.

In the following we shall assume that $(X, \mathcal{S}, m)$ is a probability space, $\Phi\colon X\Rightarrow X$ is a measurable stochastic transformation; expressions "almost every", "almost everywhere" will be connected with the measure m. Stochastic transformation Φ is called measure-preserving with the invariant measure m iff

$$m(\Phi^{-1}f)=m(f) \text{ for every } f\in M(X)\,.$$

By the same argument as in the case of measurability, stochastic transformation Φ is measure-preserving iff $m(E)=m(\Phi^{-1}E)$ for every $E\in\mathcal{S}$. In the following we shall suppose that Φ is measure-preserving, with the invariant measure m.

We define iterations of Φ in a natural way: $\Phi^1=\Phi$, $\Phi^k=\Phi\Phi^{k-1}$. Moreover, we define Φ^0 as Id_X, where Id_X is the stochastic equivalent of the ordinary identity mapping on X. The space $(X^N, \mathcal{S}^N)$ equipped with a probability P, defined consistently due to Kolmogorov theorem, such that

$$\mathrm{P}(X\times X\times\ldots\times X\times E_1\times E_2\times\ldots\times E_k\times E_{k+1}\times X\times\ldots)=$$
$$=\int_{E_1}\int_{E_2}\cdots\int_{E_k} Fx_k(E_{k+1})\ldots Fx_1(\mathrm{d}x_2)m(\mathrm{d}x_1)$$

holds, is called the space of trajectories of Φ; this construction is well-known from the literature ([1], [3], [4]). We shall use the following conventions to simplify the notation of the sets from $\mathcal{S}^N$: instead of $E_1\times\ldots\times E_k$ we shall write $(E_1, \ldots, E_k)$, instead of $E\times\ldots\times E$ (n times) we shall write nE, ∞E instead of $E\times E\times\ldots\times E\times\ldots$, P$(E)$ instead of P(nX, E, ∞X). The fact that m is an invariant measure implies that P(nX, E)=P(E); hence the preceding convention is justificated. Clearly P$(E)=m(E)$. When we denote by F^k an associated mapping with Φ^k, then the following holds:

$$\mathrm{P}(E_1, (\mathrm{n}_1-1)X, E_2, (\mathrm{n}_2-1)X, E_3, \ldots, E_{k+1})=$$
$$=\int_{E_1}\int_{E_2}\cdots\int_{E_k} F^{\mathrm{n}_k}x_k(E_{k+1})\ldots F^{\mathrm{n}_1}x_1(dx_2)\mathrm{m}(\mathrm{d}x_1)\,,$$

what can be easily seen from the corresponding definitions.

We call the set $E\in\mathcal{S}$ almost invariant, iff $\Phi^{-1}E=\chi_E$ almost everywhere. Measure-preserving stochastic transformation Φ is called ergodic, iff the measure of every almost invariant set is 0 or 1.

Now we can state a theorem of Poincaré for the stochastic transformations. In the classical theory of dynamic systems, Poincaré's theorem states, that if we have some moving incompressible fluid in a space with a finite volume, then the particle from a set with the positive volume returns to it infinitely often (cf. [4]). Our theorem states nothing but the same about stochastic transformations.

Theorem. Let $(X, \mathcal{S}, m)$ be a probability space, let $\Phi: X \Rightarrow X$ be a measure preserving stochastic transformation with invariant measure m, let $(X^N, \mathcal{S}^N, \mathrm{P})$ be a space of trajectories of Φ. If $E \in \mathcal{S}$ and $M \in \mathcal{S}^N$,

(1) $M = \{\{x_i\}_{i=0}^{\infty} \in X^N\colon\ x_0 \in E,\ x_i \in E$ for infinite number of $i \in N\}$, then $\mathrm{P}(M) = m(E)$. Moreover, if Φ is ergodic and $M' \in \mathcal{S}^N$,

$M' = \{\{x_i\}_{i=0}^{\infty} \in X^N\colon\ x_i \in E$ for infinite number of $i \in N\}$, then $\mathrm{P}(M') = 1$.

The theorem will be proved by a sequence of lemmas.

Lemma 1. Let $\Phi: X \Rightarrow X$ be a measure preserving stochastic transformation with an invariant measure m. If E does not contain an almost invariant set with nonzero measure, then for almost all $x \in E$ such n exists that $F^n x(E) < 1$.

Proof. Let $B = \{x \in E\colon F^n x(E) = 1$ for all $n = 1, 2, \ldots\}$. We shall show that B is almost invariant. Define sets N_n, $n = 1, 2, \ldots$ in the following way:

$$N_n = \{x \in E - B\colon F^i x(E) = 1 \text{ for } i < n,\ F^n x(E) < 1\}\ .$$

Clearly, $\bigcup_{n=1}^{\infty} N_n = E - B$. We shall prove that $P(B, N_n) = 0$ for all $n = 1, 2, \ldots$. Let n be fixed. If $m(N_n) = 0$ or $m(B) = 0$, the proof is completed. Suppose that $m(N_n) > 0$, $m(B) > 0$ and $\mathrm{P}(B, N_n) > 0$. Then $\int_B Fx(N_n) m(\mathrm{d}x) > 0$ and since $m(B) > 0$, then $Fx(N_n) > 0$ for all $x \in C \subset B$, $m(C) > 0$. If $m(X - E) = 0$, then

$$\int_E (Fx(E) - 1) m(\mathrm{d}x) = \int_X Fx(E) m(\mathrm{d}x) - 1 > m(E) - 1 = 0\ .$$

and then $Fx(E) = 1$ almost for every $x \in E$, therefore E is almost invariant, with measure 1, which is in contradiction with the assumption. Hence $m(X - E) > 0$. But then

$$\mathrm{P}(B, N_n, (n-1)X, X - E) = \int_B \left(\int_{N_n} Fy^n(X - E) Fx(\mathrm{d}y) \right) m(\mathrm{d}x)\ .$$

According to the assumption $F^n y(E) < 1$ for all $y \in N_n$, hence $F^n y(X - E) > 0$. Since for all $x \in C \subset B$, $m(C) > 0$, is $Fx(N_n) > 0$, then $\int_{N_n} F^n y(X - E) Fx(\mathrm{d}y) > 0$ for all $x \in C$. But then $\mathrm{P}(B, N_n, (n-1)X, X - E) > 0$, therefore $\mathrm{P}(B, nX, X - E) > 0$ and $\int_B F^n x(X - E) m(\mathrm{d}x) > 0$. Again there exists a set $A \subset B$, $m(A) > 0$, (because $m(B) > 0$) that for all $x \in A$ is $F^n x(X - E) > 0$, hence $F^n x(E) < 1$, and since A is nonvoid subset of B, we have the contradiction with the definition of B. Hence $P(B, N_n) = 0$ for all $n = 1, 2, \ldots$. Since the sets N_n are pairwise disjoint, we have $P(B, E - B) = P\left(B, \bigcup_{n=1}^{\infty} N_n\right) = \sum_{n=1}^{\infty} P(B, N_n) = 0$;

$P(B, E)=\int_B Fx(E)m(\mathrm{d}x)=\int_B 1m(\mathrm{d}x)=m(B)$ due to definition of B;
$m(B)=P(B, E)=P(B, B)+P(B, E-B)=P(B, B)$ due to additivity:

$$m(B)=\int_X Fx(B)m(\mathrm{d}x)=\int_B Fx(B)m(\mathrm{d}x)+\int_{X-B} Fx(B)m(\mathrm{d}x) = P(B, B) + \int_{X-B} Fx(B)m(\mathrm{d}x) = m(B)+\int_{X-B} Fx(B)m(\mathrm{d}x).$$

Hence $Fx(B)=0$ for almost all $x \in B$. Conversely, $0 \leqslant \int_B (1-Fx(B))m(\mathrm{d}x) = 0$, hence $Fx(B)=1$ for almost all $x \in B$. We have shown that B is almost invariant. Hence $m(B)=0$. □

Lemma 2. Let $A_1 \subset A_2 \subset \ldots \subset A_n \subset \ldots$ be a countable system of sets from $\mathscr{S}$, $A=\bigcup_{n=1}^{\infty} A_n$, $P(\infty A_n)=0$ for all $n=1, 2, \ldots$. Then $P(\infty A)=0$.

Proof. Clearly $\infty A=\bigcup_{n=1}^{\infty} \infty A_n$ and $\infty A_i \subset \infty A_j$ for $i \leqslant j$. The statement follows from the lower semicontinuity of P. □

Lemma 3. Let $A \in \mathscr{S}$ and suppose that A does not contain an almost invariant subset of nonzero measure. Then $P(\infty A)=0$.

Proof. Consider again the system N_n of sets, $n=1, 2, \ldots$ $N_n = \{x \in A: F^i x(A)=1$ for all $i<n$, $F^n x(A)<1\}$. We construct a new system $B_n = \bigcup_{i=1}^{n} N_i$. Let $B=\bigcup_{n=1}^{\infty} B_n=\bigcup_{n=1}^{\infty} N_n$, $m(B)=m(A)$(according to Lemma 1) and $B \subset A$. We shall prove that for all $n=1, 2, \ldots$ is $P(\infty B_n)=0$. Let n be fixed. For every $r=1, 2, \ldots$ there exist sets $N_1^r, \ldots, N_n^r$, such that $N_i^r \subset N_i$, $F^i x(A)<1-\frac{1}{r}$ for $x \in N_i^r$, $i=1, 2, \ldots, n$. Denote $B_n^r=\bigcup_{i=1}^{n} N_i^r$. We have $F^i x(A)<1$ for $x \in N_i$, $i=1, 2, \ldots, n$, hence $\bigcup_{r=1}^{\infty} B_n^r=\bigcup_{r=1}^{\infty}\bigcup_{i=1}^{n} N_i^r=\bigcup_{i=1}^{n}\bigcup_{r=1}^{\infty} N_i^r=\bigcup_{i=1}^{\infty} N_i=B_n$. For $x \in N_i^r$ we have $1-1/r>F^i x(A) \geqslant F^i x(N_i) \geqslant F^i x(N_i^r)$, due to corresponding inclusions and the monotonicity of the measure. Consider $\varepsilon>0$ and k positive integer, such that $k(1-1/r)^k<\varepsilon/(n(n+1)m(B_n^r))$ holds. This can be always done, because if $c=(1-1/r)^{-1}>1$, then, according to L'Hospital rule $k(1-1/r)^k=k/c^k$ tends to zero, when k tends to $+\infty$. Let $K=k(n+1)$. We shall show that for an arbitrary finite sequence $\{i_j\}_{j=1}^{K}$, $i_j \in \{1, \ldots, n\}$ for $j=1, \ldots, K$, is $P(N_{i_1}^r, \ldots, N_{i_K}^r)< (1-1/r)^k\, m(B_n^r)$, and, while we have nK of all these sequences, the subadditivity of the measure implies

$$P(B_n^r, \ldots, B_n^r)=P(KB_n^r) \leqslant nK(1-1/r)^k m(B_n^r)=n(n+1)k(1-1/r)^k m(B_n^r)<\varepsilon .$$

Hence for arbitrary $\varepsilon>0$ there exist K such that $P(KB_n^r)<\varepsilon$ and $P(\infty B_n^r) \leqslant$

$P(KB_n^r)$ for every K; hence $P(\infty B_n^r) < \varepsilon$ for every $\varepsilon > 0$; and so $P(\infty B_n^r) = 0$. We have $B_n^q \subset B_n^r$ for $q \leqslant r$, $\bigcup_{r=1}^{\infty} B_n^r = B_n$, therefore $P(\infty B_n) = 0$ due to Lemma 2. By repeated use of Lemma 2 we get $P(\infty B) = 0$. Hence $P(\infty A) = 0$, because $B \subset A$, $m(A) = m(B)$.

Now we have only to do the crucial part of the proof. Let $\{i_j\}_{j=1}^{K}$, $i_j \in \{1, \ldots, n\}$, $j = 1, \ldots, K$, is an arbitrary sequence of the length K. If $(N_{i_1}^r, \ldots, N_{i_K}^r)$ is a corresponding set from $\mathscr{S}^N$, then we consider a set

$$(N_{j_1}^r, (j_1 - 1)X, N_{j_2}^r, (j_2 - 1)X, N_{j_3}^r, \ldots, N_{j_k}^r, (j_k - 1)X, N_{j_{k+1}}^r, qX),$$

which contains the previous one and is constructed as follows:

$j_1 = i_1$
$j_2 = i_{1+j_1}$
$j_3 = i_{1+j_1+j_2}$
generally
$j_s = i_{1+j_1+j_2+\ldots+j_{s-1}}$, for $s = 1, \ldots, k$,
and $j_{k+1} = i_{1+j_1+j_2\ldots+j_k}$, $q = K - 1 - j_1 - \ldots - j_k$.

Clearly, for all $s = 1, \ldots, k+1$, $j_s \in \{1, \ldots, n\}$;

$$q = K - 1 - j_1 - \ldots - j_k = k(n+1) - kn - 1 = k - 1 \geqslant 0 .$$

Now we evaluate the measure P of this new set:

$$\begin{aligned}
&\int_{N_{j_1}^r} \cdots \int_{N_{j_{k-1}}^r} \int_{N_{j_k}^r} F^{j_k} x_k(N_{j_{k+1}}^r) F^{j_{k-1}} x_{k-1}(\mathrm{d}x_k) \ldots F^{j_2} x_2(\mathrm{d}x_3) F^{j_1} x_1(\mathrm{d}x_2) m(\mathrm{d}x_1) \leqslant \\
&\leqslant \int_{N_{j_1}^r} \cdots \int_{N_{j_{k-1}}^r} (1 - 1/r) F^{j_{k-1}} x_{k-1}(N_{j_k}^r) \ldots F^{j_1} x_1(\mathrm{d}x_2) m(\mathrm{d}x_1) \leqslant \\
&\leqslant \int_{N_{j_1}^r} \cdots \int_{N_{j_{k-2}}^r} (1 - 1/r)^2 F^{j_{k-2}} x_{k-2}(N_{j_{k-1}}^r) \ldots F^{j_1} x_1(\mathrm{d}x_2) m(\mathrm{d}x_1) \leqslant \ldots \leqslant \\
&\leqslant \int_{N_{j_1}^r} (1 - 1/r)^{k-1} F^{j_1} x_1(N_{j_2}^r) m(\mathrm{d}x) \leqslant (1 - 1/r)^k m(N_{j_1}^r) \leqslant (1 - 1/r)^k m(B_n^r) .
\end{aligned}$$

The statement holds due to the monotonicity of the measure. □

The proof of our Theorem will be based on the following weaker (but in fact equivalent) result.

Lemma 4. Let $(X, \mathscr{S}, m)$ be a probability space, Φ: $X \Rightarrow X$ a measure-preserving stochastic transformation with m as an invariant measure, let $E \in \mathscr{S}$, $(X^N, \mathscr{S}^N, P)$ be the space of trajectories, associated with the stochastic transformation Φ.

Let $M \in \mathscr{S}^N$,

$$M = \{\{x_i\}_{i=0}^{\infty} \in X^N;\ x_0 \in E,\ x_i \in E \text{ at least for one } n = 1, 2, \ldots\} .$$

Then $P(M) = m(E)$.

Proof. Suppose $A = \{x \in E\colon F^n x(E) = 1 \text{ for all } n = 1, 2, \ldots\}$. We know from

Lemma 1 that A is almost invariant and $E-A$ does not contain an invariant set with nonzero measure. Then $P(A, A)=\int_A Fx(A)m(\mathrm{d}x)=\int_A 1m(\mathrm{d}x)=m(A)$, therefore the statement holds for almost all $x\in A$ and we can restrict our considerations only to the set $E-A$ and the corresponding restriction of the stochastic transformation Φ to this set (defined in a natural way as a stochastic transformation with restricted associated mapping F); $m(A)=P(A, X)=P(A, A)+P(A, E-A)$; hence $P(A, E-A)=0$. Thus in the following, we suppose that E has not an almost invariant subset of nonzero measure. By the same argument, if B is maximal almost invariant subset of E^c, then $P(X-B, B)=0$, $P(B, B)=1$, hence we also suppose that E^c has not an almost invariant subset of nonzero measure. (Fenomenologically, trajectory gets into and from the almost invariant set with probability zero.) Applying the statement of Lemma 3 to the set E^c, we get $P(E, \infty E^c)\leqslant P(X, \infty E^c) = P(\infty E^c)=0$, hence $P(M)==P(E)-P(E, \infty E^c)=P(E)=m(E)$. □

Proof of the Theorem. Let the assumptions of the Theorem hold. Then also satisfied the assumptions of Lemma 4 are. Let M be a set defined by (1). Denote (for $n=0, 1, 2, \ldots$)

$$M_n=\{\{x_i\}_{i=0}^{\infty}\in X^N\colon x_0\in E,\ x_n\in E,\ x_i\notin E \text{ for } i>n\}\ .$$

We obtain $P(M_n)\leqslant P(E, (n-1)X, E, \infty E^c)\leqslant P(nX, E, \infty E^c)=P(E, \infty E^c)=0$. From Lemma 4 it follows that $P(E, \infty X)=P\left(M\cup\bigcup_{n=0}^{\infty}M_n\right)$. If $n>k$, then $M_n\cap M_k=\emptyset$, therefore M_n are pairwise disjoint and $P\left(\bigcup_{n=0}^{\infty}M_n\right)=\sum_{n=0}^{\infty}P(M_n)=0$. Hence $m(E)=P(E, \infty X)=P(M)$. The remainder of the statement for the ergodic stochastic transformations follows from Lemma 3. □

Remark. The assumption of ergodicity is too strong for the particular case; it is sufficient to suppose that E^c does not contain an almost invariant subset of nonzero measure.

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SÚHRN

O POINCARÉHO VETE PRE STOCHASTICKÉ INFORMÁCIE (MARKOVOVE REŤAZCE)

Ivan Mizera, Bratislava

V článku sa formuluje a dokazuje Poincarého rekurenčná veta, známa z ergodickej teórie, pre stochastické transformácie, ktoré sú zovšeobecnením obyčajných bodových zobrazení a majú úzky vzťah k Markovovým reťazcom so všeobecným stavovým priestorom.

РЕЗЮМЕ

О ТЕОРЕМЕ ПУАНКАРЕ ДЛЯ СТОХАСТИЧЕСКИХ ТРАНСФОРМАЦИЙ (ЦЕПЕЙ МАРКОВА)

Иван Мизера, Братислава

В статье формулируется и доказывается теорема Пуанкаре о возвращении. Она известна из эргодической теории, в частности для стохастических трансформаций, которые являются обобщением обычных точечных отображений и находятся в тесной связи с цепями Маркова с общим пространством состояний.

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