and

$$\text{if} \quad a \wedge b \prec b \quad \text{then} \quad (a,b)M.$$

We remark that this lemma will be needed at several places in the sequel.

In Fig. 1 we give now an example of an algebraic lattice having a pure element which is not strongly pure.

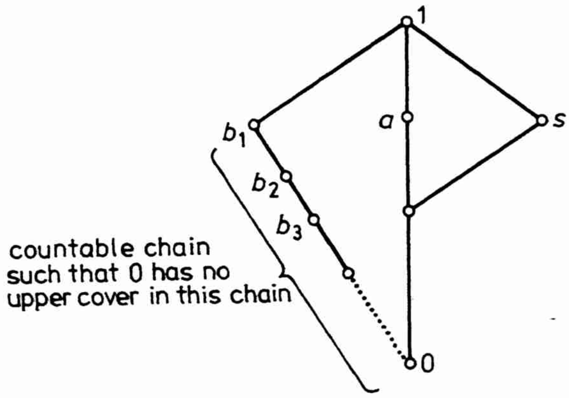

Fig. 1

This lattice is complete and satisfies the ascending chain condition. Hence every of its elements is compact (cf.[1,p.14]). It is easily checked that the element a is pure. But the element a is not strongly pure. To see this, take the compact element s. We have

$$a \vee s = a \vee b_i \quad \text{and} \quad a \wedge b_i = 0 \ (i=1,2,3,\dots) \ .$$

Moreover, we have $a \prec a \vee b_i$. Now by the preceding Lemma $(a,b_i)M$ would imply the relation $a \wedge b_i = 0 \prec b_i$ which, however, does not hold for any i $(i=1,2,3,\dots)$.