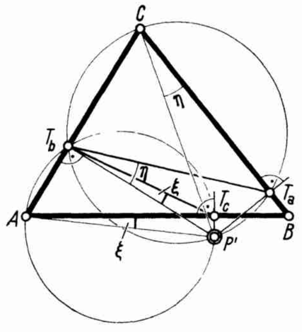
Die Punkte P', A, T_b, T_c und P', C, T_a, T_b liegen nach dem Satz von Thales je auf einem Kreis (siehe Figur 2). Bei allgemeiner Wahl von P' ergeben sich aus dem Peripheriewinkelsatz folgende gleiche Winkel: $\xi = \sphericalangle\, P'AB = \sphericalangle\, P'T_bT_c$, $\eta =$



Figur 2

$\sphericalangle\, P'CB = \sphericalangle\, P'T_bT_a$. Also unterscheiden sich ξ und η um $\sphericalangle\, T_aT_bT_c$. Genau dann, wenn T_a, T_b, T_c auf einer Geraden liegen, müssen ξ und η gleich sein, daher die vier Punkte A, B, C, P' auf einem Kreis liegen. Also: *Dann und nur dann liegen T_a, T_b, T_c auf einer Geraden, wenn P' auf den Umkreis des Dreiecks $\triangle$ fällt.*

In Verbindung mit der Deutung von (4) erhält man das Ergebnis: *Die gefährliche Fläche beim räumlichen Rückwärtsschnitt ist jener Drehzylinder, der die Ebene ε nach dem Umkreis des Dreiecks $\triangle$ schneidet.*

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On a Diophantine Equation

A. SCHINZEL and W. SIERPIŃSKI [2][1]) have recently showed that all solutions of the equation

$$(x^2-1)(y^2-1) = \left(\left(\frac{y-x}{2}\right)^2 - 1\right)^2 \tag{1}$$

in natural numbers x, y, $x < y$ are of the form $x = x_n$, $y = x_{n+1}$, $n = 0, 1, \ldots$, where $x_0 = 1$, $x_1 = 3$, $x_{n+2} = 6\,x_{n+1} - x_n$. Equation (1) is a special case of the equation

$$(x^2-1)(y^2-1) = (z^2-1)^2 \tag{2}$$

whose all solutions are still unknown (cf. [2]–[5]). Moreover, the only known solution of (2) which is not a solution of (1) is $x = 4$, $y = 31$, $z = 11$ ([5]; cf. [2], [3]).

The purpose of this note is to give all solutions in natural numbers t, x, y of the equation

$$(x^2-t^2)(y^2-t^2) = \left(\left(\frac{y-x}{2}\right)^2 - t^2\right)^2. \tag{3}$$

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[1]) Numbers in brackets refer to References, page 38.

We shall prove the following

Theorem. All solutions of the Equation (3) in distinct natural numbers $t, x, y, x < y$, are of the form

$$t = |\, m^2 - 2\, n^2 \,|\, k \,, \quad x = (m^2 + 2\, n^2)\, k \,, \quad y = (3\, m^2 + 8\, m\, n + 6\, n^2)\, k \,, \tag{4}$$

where m, n, k are natural numbers.

Proof. As in [2] we observe that

$$(x^2 - t^2)\,(y^2 - t^2) - \left(\left(\frac{y-x}{2}\right)^2 - t^2\right)^2 = -\frac{1}{16}\,(x+y)^2\,(x^2 - 6\,x\,y + y^2 + 8\,t^2)\,.$$

Thus (3) is equivalent to

$$x^2 - 6\,x\,y + y^2 + 8\,t^2 = 0\,,$$

which may be written in the form

$$(y - 3\,x)^2 = 8\,(x^2 - t^2)\,. \tag{5}$$

First of all, $t < x$. Further, from (5) it follows that $4 \mid y - 3\,x$, so $y - 3\,x = 4\,z$, where $z > 0$, since otherwise $(y - x)/2 \leqslant x$ and from $t < x < y$ and (3) we get $(x^2 - t^2)\,(y^2 - t^2) \leqslant (x^2 - t^2)^2$, $y \leqslant x$, which is impossible.

Thus $z > 0$ and (5) can be written as

$$2\,z^2 + t^2 = x^2\,. \tag{6}$$

Consequently, every solution of (5) in natural numbers $t, x, y, x < y$, gives a solution of (6) in natural numbers t, x, z, where $4\,z = y - 3\,x$. On the other hand, if t, x, z is a solution of (6) in natural numbers, then the numbers $t, x, y = 3\,x + 4\,z$ are natural, $x < y$, and they form a solution of (5).

Thus, in order to find all solutions of (3) in natural numbers $t, x, y, x < y$ it suffices to know all solutions of (6) in natural numbers t, x, z and put $y = 3\,x + 4\,z$.

But all solutions of (6) are the following (cf. [1], p. 41):

$$t = |\, m^2 - 2\, n^2 \,|\, k \,, \quad x = (m^2 + 2\, n^2)\, k \,, \quad z = 2\, m\, n\, k \,,$$

where m, n, k are natural numbers. If we put here $y = 3\,x + 4\,z$, we get the formulae (4), which completes the proof.

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