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Titel: Banach-Stone Theorems and Separating Maps.

Autor: Hernandez, S.; Beckenstein, E.

Jahr: 1995

PURL: https://resolver.sub.uni-goettingen.de/purl?365956996_0086|log32

Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

info@digizeitschriften.de

manuscripta math. 86, 409 - 416 (1995)

manuscripta mathematica
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Banach-Stone Theorems and Separating Maps*

S. Hernandez[†], E. Beckenstein and L. Narici

Universitat Jaume I, Departamento de Matemáticas, Campus de Penyeta Roja, 12071 Castellón, Spain E-mail address: hernande@llevant.uji.es.

Department of Mathematics, St. John's University, Staten Island, NY 10301, USA E-mail address: beckenst@sjuvm.stjohns.edu.

Department of Mathematics, St. John's University, Jamaica, NY 11439, USA E-mail address: naricil@sjuvm.stjohns.edu.

Received May 4, 1993

Let $C(X,E)$ and $C(Y,F)$ denote the spaces of continuous functions on the Tihonov spaces X and Y, taking values in the Banach spaces E and F, respectively. A linear map $H : C(X,E) \to C(Y,F)$ is *separating* if $f(x)g(x) = 0$ for all x in X implies $Hf(y)Hg(y) = 0$ for all y in Y. Some automatic continuity properties and Banach-Stone type theorems (i.e., asserting that isometries must be of a certain form) for separating maps H between spaces of real- and complex-valued functions have already been developed. The extension of such results to spaces of vector-valued functions is the general subject of this paper. We prove in Theorem 4.1, for example, for compact X and Y, that a linear isometry H between $C(X,E)$ and $C(Y,F)$ is a "Banach-Stone" map if and only if H is "biseparating (i.e, H and H^{-1} are separating). The Banach-Stone theorems of Jerison and Lau for vector-valued functions are then deduced in Corollaries 4.3 and 4.4 for the cases when E and F or their topological duals, respectively, are strictly convex.

1 Introduction

$C(X,E)$ and $C(Y,F)$ always carry their respective compact-open topologies. If $E = R$ or C, we write simply $C(X)$, etc. $C^*(X,E)$ denotes the subspace of continuous functions which have precompact range endowed with its sup norm topology: $\|f\| = sup_{x\in X} |f(x)|$. The locally convex properties (bornologicity,barreledness, etc.) of $C(X,E)$ with compact-open topology may be found in [19], while the general topic of Banach-Stone theorems for vector-valued functions is elaborated in [10] and [11].

In the context of real- or complex-valued functions, separating maps were introduced by Beckenstein and Narici in [7]-[9] and further investigated in [1], [2], [7], [8], [9],and [15]. In many cases, separating maps are automatically continuous (cf. [5], [6]): Separating isomorphisms between spaces of *bounded* continuous

*1991 Mathematics Subject Classification: 46B04, 46E40, 46E15

Key words and phrases: Banach-Stone theorem, automatic continuity, separating map

[†]Research supported by the Fundació Caixa Castelló, MI/25.043/92

functions are automatically continuous, for example. In the context of ordered spaces, similar automatic continuity questions have been considered in [3]–[6] and [18].

We establish some automatic continuity properties of separating maps in Secs. 2 and 3. In 3.4 we show, under rather general conditions, that if a separating map H takes nonvanishing functions of $C(X,E)$ into nonvanishing functions of $C(Y,F)$, then it must be continuous and of the "Banach-Stone form." As an application, we show in Theorem 4.1 that if X and Y are compact and $H{:}C(X,E) \to C(Y,F)$ is a separating linear isometry then H is a strong Banach-Stone map (see Example 3.1 for the definition) if and only if H is "biseparating" in the sense that H and H^{-1} are separating. The Banach-Stone theorems of Jerison and Lau are then deduced in 4.2 and 4.3 for the cases when E and F or their topological duals, respectively, are strictly convex.

We adhere to the following notational conventions. For $f \in C(X,E)$, $Z(f)$ and $coz(f)$ denote the zero set and cozero set of f, respectively. $L(E,F)$ denotes the set of all continuous linear maps of E into F endowed with the strong operator topology, as defined in [12] p. 475. For $e \in E$, $\tilde{e}$ denotes the constant function which maps each element in X into e. $\tilde{E}$ denotes the subspace of $C^*(X,E)$ consisting of all such constant functions.

2 Vector-valued separating maps

For $f \in C^*(X)$ or $C(X)$, βf and υf denote the continuous extensions of f to the Stone-Cech compactification βX and realcompactification υX of X, respectively. M denotes a $C^*(X)$-module of $C(X,E)$. A linear map $H : M \to F$ is separating if, for all $f,g \in M$, $\| f(x)g(x) \| = 0$ for all $x \in X$ implies that $\| HfHg \| = 0$. Evaluation maps, $f \longmapsto f(x)$, or $f \longmapsto f(g(x))$, where $g : Y \to X$ is continuous, for example, are separating. A point $p \in X$ is a *weak support* for the separating map $H : M \to F$ if for any $f \in M$ and any open neighborhood U of p in βX, $f = 0$ on $U \cap X$ implies that $Hf = 0$. A point $p \in \beta X$ is a *support* for H if, for any $f \in C^*(X,E)$, (resp. $C(X,E)$), $\beta f(p) = 0$ (resp. $\upsilon f(p) = 0$) implies $Hf = 0$; when p is a support for f it follows that $Hf = H(\beta f(p)\tilde{\ })$ (resp. $H(\upsilon f(p)\tilde{\ })$ for any $f \in C^*(X,E)$ (resp. $C(X(E))$).

By arguments analogous to those in [9] and [1] for the case of real- or complex-valued functions, 2.1 and 2.2 follow.

Theorem 2.1 *If $H : M \to F$ is separating, then there is a unique weak support $p_H \in \beta X$ for H.*

Corollary 2.2 *If the separating map $H : M \to F$ is continuous then $p_H \in X$ and is a support for H.*

Theorems 2.3 and 2.4 say essentially that a separating map which maps nonvanishing functions into nonvanishing functions is automatically continuous.

Theorem 2.3 *The separating map $H{:}C^*(X,E) \to F$ is continuous if the following conditions are satisfied:*

(1) For all $f \in C^*(X,E)$ with $\|f(x)\| \geq r > 0$ for all $x \in X$, we have that $H(f) \neq 0$;

(2) $H(\tilde{E}) \supset H(C^*(X,E))$;

(3) $H|_{\tilde{E}}$ is continuous.

When $H|_{\tilde{E}}$ is injective, the converse holds.

Proof. If H is continuous then, by 2.2, it follows that $p_H \in X$ and is a support for H so $Hf = H(f(p_H)\tilde{\ })$, from which (1) and (2) follow; (3) is clear.

Conversely, suppose that (1)–(3) are satisfied. First, we prove that the weak support p_H is a support for H. If not, there is $f \in C^*(X,E)$ such that $\beta f(p_H) = 0$ and $Hf = d_0 \neq 0$. By (2), there is $e_0 \in E$ such that $H(\tilde{e_0}) = d_0$. Let r be such that $\|e_0\| > r > 0$ and let $h : E \to E$ be defined as $h(e) = e$ when $\|e\| \leq r$, and $h(e) = re/\|e\|$ when $\|e\| > r$. The map h is continuous and coincides with the identity map on $\{e \in E : \|e\| < r\}$. For $g = h \circ f - \tilde{e_0}$,note that $g \in C^*(X,E)$ and that g coincides with f on the open neighborhood $\{p \in \beta X : \|\beta f(p)\| < r\}$ of p_H. Thus,$Hg = H(f - \tilde{e_0}) = 0$. On the other hand, $\|g(x)\| \geq \|e_0\| - r > 0$ for all x in X. As this contradicts (1), it follows that p_H is a support for H so $Hf = H(\beta f(p_H)\tilde{\ })$ for all $f \in C^*(X,E)$. To see that H is continuous, let (f_n) be a sequence in $C^*(X,E)$ which converges to f. As such, $\beta f_n \to \beta f$ so $\beta f_n(p_H) \to \beta f(p_H)$. Since $H|_{\tilde{E}}$ is continuous, $H(\beta f_n(p_H)\tilde{\ }) = Hf_n$ converges to $H(\beta f(p_H)\tilde{\ }) = Hf$.

Upon leaving $C^*(X,E)$ for $C(X,E)$, but for realcompact X, we have:

Theorem 2.4 *If X is realcompact then the separating map $H : C(X,E) \to F$ is continuous if the following conditions are satisfied:*

(1) for all nonvanishing $f \in C(X,E)$, $Hf \neq 0$;

(2) $H(\tilde{E}) \supset H(C(X,E))$;

(3) $H|_{\tilde{E}}$ is continuous.

If $H|_{\tilde{E}}$ is injective, then the converse holds, even if X is not realcompact.

Proof. If H is continuous then, by 2.2, $p_H \in X$ and is a support for H. From this, (1)–(3) are easily verified.

Conversely, suppose that (1)–(3) are satisfied. By 2.1, there exists a weak support $p_H \in \beta X$ for H. We prove that $p_H \in X$ and is a support for H. If p_H is not in the realcompact space X then there exists a sequence (U_n) of neighborhoods of p_H in X whose intersection does not meet X. For each n, choose neighborhoods W_n and V_n of p_H in X such that $W_n \supset cl_\beta V_n \supset V_n \supset cl_\beta W_{n+1} \supset W_{n+1}$. By the Tietze extension theorem (on βX), for each positive integer n, there exists $h_n \in C(X)$ such that $0 \leq h_n \leq 1$, $h_n(X \backslash W_n) = \{0\}$, $h_n(clV_n \backslash W_{n+2}) = \{1\}$ and $h_n(V_{n+2}) = \{0\}$. In addition, define $h_0 \in C(X)$ such that $0 \leq h_0 \leq 1$, $h_0(V_3) = \{0\}$, and $h_0(X \backslash W_2) = \{1\}$. The series $h = \sum_{n \geq 0} 2^{-n} h_n$ converges uniformly and therefore belongs to $C(X)$. Let e be a unit vector in E and let $g = h \cdot e$, i.e., for each x X, $g(x) = \sum_{n \geq 0} 2^{-n} h_n(x)e$. Evidently $g(x) \in C(X,E)$. Since h never vanishes, neither does g. Therefore $Hg \neq 0$ by (1). By (2) there exists $u \in E$ such that $Hg = Hu\tilde{\ }$. For any positive integer n, by the way h_n is defined, $h_n e$ vanishes on the neighborhood V_{n+2} of p_H; since H is separating, it follows that $H(h_n e) = 0$. Thus, $Hu\tilde{\ } = H(\sum_{n \geq N} 2^{-n} h_n e)$ for any N. But $\sum_{n \geq N} 2^{-n} h_n(x)e$ converges uniformly to 0 so if N is chosen such that

$\left\|\sum_{n\geq N} 2^{-n}h_n(x)e\right\| < \|u\|$ for every x in X, then $H(\sum_{n\geq N} 2^{-n}h_n(x)e - u^{\sim}) = 0$ even though $\sum_{n\geq N} 2^{-n}h_n(x)e - u^{\sim}$ never vanishes; this contradicts (1). Therefore $p_H \in X$. It now follows exactly as in 2.3 that p_H is a support for H and that H is continuous.

Remark 2.5 *If X is not realcompact, then (1)–(3) of 2.4 imply that $p_H \in X$.*

3 Separating Linear Operators

Now consider the more general situation, that $H{:}C(X,E) \to C(Y,F)$ is separating.

Example 3.1 *If $J : Y \to L(E,F)$ and $g : Y \to X$ are continuous then the map H defined at each f in $C(X,E)$ and y in Y as $Hf(y) = (Jy)[f(g(y))^{\sim}]$ is separating. We refer to maps of this type as Banach-Stone maps. If g is a homeomorphism and Jy is an isometry for each y in Y, then H is called a strong Banach-Stone map. (Cf. [10], 8.2(ii), p.142).*

After defining the *support map* and the *associated map* of a separating map, we show that *continuous* separating maps are Banach-Stone maps in Theorem 3.3.

For $y \in Y$, let $\hat{y}$ denote the evaluation map $\hat{y}{:}C(Y,F) \to F$, $f \mapsto f(y)$. As a composite of separating maps, $\hat{y} \circ H$ is separating and it is straightforward to adapt the argument of [9], Prop.2.1(a) for real- or complex-valued functions to the vector-valued situation to get:

Theorem 3.2 *Let $H{:}C(X,E) \to C(Y,F)$ be separating and let $D = \cup\{coz(Hf) : f \in C(X,E)\}$. Then the support map of H, the map $h : D \to \beta X$ sending $y \in D$ into the weak support $p_{\hat{y}\circ H}$ of $\hat{y} \circ H$, is continuous.*

Continuity of a separating map H is characterized by means of the continuity of the maps $\hat{y} \circ H$ in Theorem 3.3. First, we need the following lemma.

Lemma 3.3 *Let $H{:}C(X,E) \to C(Y,F)$ be separating and let $D = \cup\{coz(Hf) : f \in C(X,E)\}$. If $\hat{y} \circ H$ is continuous for all $y \in D$ then the associated map $\hat{H} : D \to L(E,F)$ of H, defined by taking $\hat{H}y(e) = (\hat{y} \circ H)(\tilde{e})$ for all $y \in D$ and $e \in E$, is continuous.*

Proof. As the compact-open topology on $\hat{E}$ coincides with the norm topology in E, it is clear that $\hat{H}y \in L(E,F)$ for all $y \in D$. As to the continuity of $\hat{H}$, let the net y_i converge to y in D. For any $e \in E$, $\hat{H}y_i(e) = H\tilde{e}(y_i) \to H\tilde{e}(y) = \hat{H}y(e)$.

The associated map $\hat{H} : D \to L(E,F)$ of 3.3 is just the adjoint of H restricted to $\hat{D} = \{\hat{y} : y \in D\}$ applied to the subspace $\tilde{E}$ of $C(X,E)$.

We now show that continuous separating maps are Banach-Stone maps.

Theorem 3.4 *Let $H{:}C(X,E) \to C(Y,F)$ be separating and let $D = \cup\{coz(Hf) : f \in C(X,E)\} = Y$. Then: (a) H is continuous if and only if $\hat{y} \circ H$ is continuous for all $y \in Y$. (b) When H is continuous then $Hf(y) = (\hat{H}y)[f(h(y))^{\sim}]$ for all y in Y and f in $C(X,E)$ where h and $\hat{H}$ are as in Theorem 3.2 and Lemma 3.3, respectively.*

Proof. Necessity is obvious. To prove sufficiency, suppose the net $f_i \to f$ in $C(X,E)$ and let K be a compact subset of Y. For any y in K and e in E, we have that $\left\|\hat{H}y\,(e)\right\| = \|H\tilde{e}\,(y)\| \leq \sup\{\|xHe\,(k)\| : k \in K\}$. The family $\{\hat{H}y : y \in K\}$ is therefore pointwise bounded on the Banach space E, hence uniformly bounded, i.e., $\left\|\hat{H}y\right\| \leq M$, for some real constant M and all y in K. For any y in K,

$$\begin{aligned} \|Hf_i(y) - Hf(y)\| &= \left\|(\hat{y} \circ H)(f_i(h(y)^{\tilde{}}) - (\hat{y} \circ H)(f(h(y)^{\tilde{}})\right\| \\ &= \left\|\hat{H}y(fi(h(y)) - \hat{H}y(f(h(y))\right\| \\ &\leq \left\|\hat{H}y\right\| \|fi(h(y)) - (f(h(y))\| \\ &\leq M \sup\{\|fi(h(k) - f(h(k))\| : k \in K\} \end{aligned}$$

Since y is an arbitrary element of K, the continuity of H follows.

(b) For continuous H, by Theorems 2.4, 3.2 and 3.3, it follows that $Hf(y) = (\hat{y} \circ H)(f) = (\hat{y} \circ H)(f(h(y)^{\tilde{}}) = \hat{H}y(f(h(y)^{\tilde{}})$ for all $y \in Y$ and $f \in C(X,E)$.

The result below now follows from Theorems 2.4 and 3.4.

Corollary 3.5 *If H:$C(X,E) \to C(Y,F)$ is separating, X is realcompact, $D = Y$ and $\hat{H}$ is as in 3.3 then H is continuous if the following conditions hold:*

(1) H sends nonvanishing functions of $C(X,E)$ into nonvanishing functions of $C(Y,F)$;

(2) for all y Y and $f \in C(X,E)$ there exists $e \in E$ such that $\hat{H}y(e) = Hf(y)$;

(3) $\hat{H}(Y) \subset L(E,F)$;

If $\hat{H}y$ is injective for all $y \in Y$ then the converse holds.

Remark 3.6 *Notice that (1) is the only relevant assertion when E and F coincide with $\mathbf{R}$ or $\mathbf{C}$ in Theorems 2.3, 2.4 and Corollary 3.5*

4 Banach-Stone Theorems

The study of when linear isometries between spaces of continuous vector-valued functions are Banach-Stone maps was initiated by Jerison in [16] and elaborated upon by many others, e.g., [10],[11],[14],[16],[17]. Among other things, we deduce a result of Jerison's by means of the techniques of separating maps in Corollary 4.2.$I(E,F)$ denotes the subspace of $L(E,F)$ of isometries. A surjective linear isomorphism H:$C(X,E) \to C(Y,F)$ is called *biseparating* if H and H^{-1} are separating.

Theorem 4.1 *Let X and Y be compact. A surjective linear isometry H:$C(X,E) \to C(Y,F)$ is a strong Banach-Stone map if and only if H is biseparating.*

Proof. If H is biseparating then H is Banach-Stone map by 3.4: $Hf(y) = (\hat{H}y)[f(h(y))^{\tilde{}}]$ for all $y \in Y$ and $f \in C(X,E)$. It remains to show that $\hat{H}y \in I(E,F)$for all y and that h is a homeomorphism. To prove that $\hat{H}y \in I(E,F)$ for every y, we first show that H maps nonvanishing functions into nonvanishing functions. To this end, let $f \in C(X,E)$ be nonvanishing and suppose that $y \in Y$

is such that $Hf(y) = 0$. Since X is compact, $0 < m = \inf\{\|f(x)\| : x \in X\}$. Let $U = \{z \in Y : \|Hf(z)\| \leq m/3\}$ and $V = \{z \in Y : \|Hf(z)\| \geq 2m/3\}$. There exists $u \in C(Y)$ such that $0 \leq u \leq 1$ with $u(U) = \{1\}$ and $u(V) = \{0\}$. Let $g = u \cdot Hf$ and note that $\|g\| \leq 2m/3$. Let $k : X \to Y$ be the support map of H^{-1}. As H^{-1} is 1-1, k must be onto because of the form H^{-1} must have (3.4), so we may choose $x \in X$ such that $k(x) = y$, where y is as above. Hf and g coincide in the neighborhood U of $k(x)$, so $H^{-1}g(x) = (H^{-1}Hf)(x) = f(x)$; hence $\|H^{-1}g\| \geq m$ which is contradictory because H^{-1} is an isometry and $\|g\| \leq 2m/3$. It follows that H maps nonvanishing functions into nonvanishing functions. If, for some y, $\hat{H}y$ is not an isometry then for some unit vector e in E, $\left\|\hat{H}y(e)\right\| = \|H\tilde{e}(y)\| < 1$. Let $H\tilde{e}(y) = d$. As H is surjective, there exists $g \in C(X, E)$ such that $Hg = \tilde{d}$. Thus $g < 1$ and $\tilde{e} - g$ never vanishes on X. As $H(\tilde{e} - g)(y) = 0$, this is contradictory. To see that the support map h is a surjective homeomorphism, note that h must be onto because H is 1-1 and Y is compact. To see that h is injective, let u and v be distinct points of Y and $g \in C(Y, F)$ be such that $g(u) \neq 0$ and $g(v) = 0$. Choose $f \in C(X, E)$ such that $Hf = g$. Thus, $0 \neq Hf(u) = (\hat{H}u)[f(h(u))\tilde{\ }]$ while $0 = Hf(v) = (\hat{H}v)[f(h(v))\tilde{\ }]$. As $\hat{H}v$ is an isometry, it follows that $0 = f(h(v)) = 0 \neq f(h(u))$ so $h(v) \neq h(u)$.

Conversely, suppose that H is a strong Banach-Stone map: $Hf(y) = (Jy)[f(g(y))\tilde{\ }]$ for all y in Y and f in $C(X, E)$, where $J : Y \to I(E, F)$ is continuous and $g : Y \to X$ is a homeomorphism. Evidently, H is a separating isometry. Since $Jy \in I(E, F)$ for all y, we may now define the mapping $K{:}X \to I(F, E)$ by taking at $x \in X$, $Kx(d) = J(g^{-1}(x))(d)$ for all $d \in F$. Since g is a homeomorphism it is easy to verify that $H^{-1}f(x) = (Kx)[f(g^{-1}(x))\tilde{\ }]$ for all $x \in X$ and $f \in C(Y, F)$ and anything of this form is separating.

Example 4.2 *Theorem 4.1 fails if H is only a separating isometry. To see this, let $X = \{0\}$, $E = R^2$ with sup norm, $Y = \{0, 1\}$ and $F = R$ with its usual norm. The surjective separating isometry $H{:}C(X, E) \to C(Y, F)$ defined as $H(a, b) = g$ with $g(0) = a$ and $g(1) = b$ is not biseparating and h is not 1-1.*

Our strategy in Corollaries 4.3 and 4.4 is to show that the conditions imply that the isometry is biseparating and then apply Theorem 4.1.

Corollary 4.3 *([16]) If X and Y are compact and E and F are strictly convex then a surjective linear isometry $H{:}C(X, E) \to C(Y, F)$ is a strong Banach-Stone map.*

Proof. We prove only that H is separating, as a similar argument proves that H^{-1} is separating. Let $f, g \in C(X, E)$ be norm-one functions with disjoint cozero sets and suppose that $y \in Y$ is such that $Hf(y) = a \neq 0$ and $Hg(y) = b \neq 0$; further assume that $\|a\| \geq \|b\|$. Since F is strictly convex, if a is not a scalar multiple of b, then $\|a\| < (\|a + b\| + \|a - b\|)/2$; hence $\|a\| < \max(\|a + b\|, \|a - b\|)$. If a is a multiple of b then clearly $\|a\| < \max(\|a + b\|, \|a - b\|)$. Let us suppose that $\|a + b\| > \|a\| \geq \|b\|$ and let d be a positive number such that $d < (1/2)(\|a + b\| - \|a\|)$. Define $V = \{y \in Y : \|Hf(y)\| < \|a\| + d\}$, $W = \{y \in Y : \|Hg(y)\| < \|b\| + d\}$ and $U = V \cap W$. U is an open neighborhood

of y. By complete regularity, there is $h \in C(Y,F)$ such that $0 \leq \|h\| \leq 1$ with $h(y) = (a+b)/\|a+b\|$ and $h(Y \backslash U) = 0$. Thus,

$$\|Hf(y) + Hg(y) + h(y)\| = \|a+b+(a+b)/\|a+b\|\| = 1 + \|a+b\|$$

from which it follows that $\|Hf + Hg + h(y)\| \geq 1 + \|a+b\|$. Let $k \in C(X,E)$ be such that $Hk = h$ so $\|f+g+k\| \geq 1 + \|a+b\|$. Since f and g have disjoint cozero sets, $\max(\|f+k\|, \|g+k\|) \geq 1 + \|a+b\|$. Thus, if we assume that $\|f+k\| \geq 1+\|a+b\|$ then $\|Hf+h\| \geq 1+\|a+b\|$. For an arbitrary element z of Y, if $z \in U$ then $\|Hf(z)+h(z)\| \leq \|Hf(z)\|+\|h(z)\| < \|a\|+d+1 < 1+\|a+b\|$. If z is not in U then $\|Hf(z)+h(z)\| \leq \|Hf(z)\|+\|h(z)\| \leq 1$. This contradiction proves that H is separating and the desired result now follows from Theorem 4.1.

Corollary 4.4 *([17]) With the same hypothesis as in Corollary 4.3 but with the topological duals of E and F being strictly convex instead of E and F, then H is a strong Banach-Stone map.*

Proof. We prove only that H is separating as the same argument shows that H^{-1} is separating. If H is not separating then there are $f, g \in C(X,E)$ with disjoint cozero sets such that $Hf(y0) = a \neq 0$ and $Hg(y0) = b \neq 0$ at some $y_0 \in Y$. Let $v \in F'$, the topological dual of F, and define $d(y0,v) \in C(Y,F)'$ by taking $d(y_0,v)(h) = v(h(y_0))$ for all $h \in C(Y,F)$. Let $\hat{H}$:$C(Y,F)' \to C(X,E)'$ denote the adjoint of H. For all $y \in Y$ and $v \in F'$ there are $x \in X$ and $u \in E'$ such that $\hat{H}(d(y,v)) = d(x,u)$ by [17, Lemma 2 (i)]; similarly, for $y \in Y$ and $\tilde{v} \in F'$ there are $\tilde{x} \in X$ and $\tilde{u} \in E'$ such that $\hat{H}(d(y,\tilde{v})) = d(\tilde{x},\tilde{u})$. By [17, Lemma 2 (ii)], $x = \tilde{x}$. Take $v_1, v_2 \in F'$ such that $\|v_1\| = \|v_2\| = 1$, $v_1(a) = \|a\|$, $v_2(b) = \|b\|$ and consider $d(y_0,v1)$ and $d(y_0,v2)$. There are $x_0 \in X$ and $u_1, u_2 \in E'$ such that $\hat{H}(d(y_0,v_i) = d(y_0,u_i)$ for $i = 1,2$. Thus, $u_1(f((x_0)) = d(x_0,u_1)(f) = d(y_0,v_1)(Hf) = v_1(a) = \|a\| \neq 0$. Therefore $f(x_0) \neq 0$; similarly, $g(y_0) \neq 0$. This contradiction completes the proof.

References

[1] J. Araújo, E. Beckenstein and L. Narici, *Biseparating maps and rings of continuous functions*, Proceedings of the Wighth Summer Conference on Topology, Queens College,New York, June 1992, Annals of the New York Academy of Sciences 728, 296-303 (1994)

[2] J. Araújo .,E. Beckenstein, and Narici, L.: Bispeparating maps and homeomorphic realcompactifications, J. Math. Anal. App., to appear.

[3] Abramovic, Y.: Multiplicative representations of disjointness preserving operators, Indag. Math. 45, 265-279 (1983)

[4] Abramovic, Y., Veksler, A., and Koldunov, V.: On operators preserving disjointness, Soviet Math. Dokl. 20, 1089-1093 (1979)

[5] Albrecht, E. and Neumann, M.: Automatic continuity of generalized local linear operators, Manuscripta Math. 32, 263-294 (1980)

[6] Albrecht, E. and Neumann, M.: Automatic continuity for operators of local type. (Lecture Notes in Mathematics 975), 342-355, Berlin-Heidelberg-New York : Springer (1985)

[7] Beckenstein, E. and Narici, L.: A nonarchimedean Stone-Banach theorem, Proc. A. M. S. 100, 242-246 (1987)

[8] E. Beckenstein and L. Narici, *Automatic continuity of certain linear isomorphisms.* Acad. Royale Belg. Bull. Cl. Sci. 73(5) , 191-200 (1987)

[9] Beckenstein, E. and Narici, L.: Automatic continuity of linear maps of spaces of continuous functions. Manuscr. Math. 62, 257-275 (1988)

[10] Behrends, E.: M-Structure and the Banach-Stone theorem (Lecture notes in mathematics 736) Berlin-Heidelberg-New York: Springer (1979)

[11] Behrends, E. and Schmidt-Bichler, V.: M-structure and the Banach-Stone theorem, Studia Math. 69, 33-40 (1980)

[12] Dunford, N. and Schwartz, J.: Linear Operators, Part I. New York: Interscience (1957)

[13] Gillman, L. and Jerison, M.: Rings of continuous functions, Princeton: Van Nostrand (1960)

[14] Jarosz, K.: Isometries between injective tensor products of Banach spaces, Pacific J. Math. 121, 383-395 (1986)

[15] Jarosz, K.: Automatic continuity of separating linear isomorphisms, Canad. Math. Bull. 33, 139-144 (1990)

[16] Jerison, M.: The space of bounded maps into a Banach space, Ann. of Math. 52, 309-327 (1950)

[17] Lau, K.: A representation theorem for isometries of $C(X, E)$, Pacific J. Math. 60, 229-232 (1975)

[18] De Pagter, B.: A note on disjointness preserving operators, Proc. A. M. S. 90, 543-549 (1984)

[19] Schmets, J.: Spaces of vector-valued continuous functions. (Lecture notes in mathematics 1003) Berlin-Heidelberg-New York (1983)