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The Kiev Algorithm for bocses applies directly to representation-finite algebras

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0. Introduction.– Let k be an algebraically closed field and Λ a finite dimensional k-algebra. It is known (see [D] and [CB;6]) that, from the representation theory point of view, the study of modΛ —the category of finite-dimensional Λ-modules —is reduced to the study of the category of representations of an additive Roiter bocs —a particular case of a layered bocs (see [CB;3.5,3.6] for the definition of these notions). This reduction is indirect since one must first consider the representation equivalent category $P_1(\Lambda)$ —of maps between finite dimensional projective Λ-modules, with image contained in the radical of the codomain.

On the other hand, an algorithm has been proposed to produce inductively, from a given representation-finite additive Roiter bocs $\mathcal{A}$, a finite sequence $\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_s$ of bocses of the same type such that : $\mathcal{A} = \mathcal{A}_1, \mathcal{A}_s$ is trivial, and they all have equivalent categories of representations (see [R-K] and [R]). The basic steps of this algorithm have been reformulated, enriched and applied successfully to more general situations (see again [R], [D], and [CB]).

In this note we show that some form of this algorithm can be applied to a wide class of representation-finite bocses "with relations" (see Theorem 7). This class contains any representation-finite k-algebra Λ, when considered as a bocs with the trivial bocs structure (namely the principal bocs associated with Λ). Thus we obtain directly a trivial bocs whose category of representations is equivalent to modΛ.

Throughout this note, unless otherwise specified, we follow the terminology and conventions of [CB]. Given a bocs $\mathcal{A} = (A, V)$ and a functor $\theta : A \longrightarrow B$, we shall abuse of the language denoting the induced morphism of bimodules $(\theta, \theta_1) : \mathcal{A} \longrightarrow \mathcal{A}^\theta$ with the same letter θ (see [CB;3]).

1.– Given a bocs $\mathcal{A} = $ (A,V) and any ideal I of A, the canonical projection $\eta : A \longrightarrow A/I$ induces a bocs $\mathcal{A}^\eta$ over the quotient category A/I, which we often denote by $\mathcal{A}_I$. It is known —see [CB;3.1]— that the category $\mathfrak{R}(\mathcal{A}^\eta)$ is identified through η^* with the full subcategory of $\mathfrak{R}(\mathcal{A})$ formed by the representations which vanish on I.

If $\mathcal{A}$ has a grouplike ω with associated differential δ, we call an ideal I of A *compatible with* ω (or *with* δ), iff I admits a finite generating set $b_1, b_2, ..., b_t$ as an A–A-bimodule such that $\delta(b_i) \in I_{i-1}V + VI_{i-1}$, for all $0 < i \leq t$ where I_i denotes the A-A-subbimodule of I generated by $b_1, b_2, ..., b_i$, for $0 \leq i \leq t$. As usual, if A has direct sums, we require that all b_i are indecomposable.

Lemma.– Assume $\mathcal{A} = (A, V)$ is a bocs with grouplike ω and let I be any ideal of A compatible with ω. Then $\mathfrak{R}(\mathcal{A}_I)$ is *isoclosed* in $\mathfrak{R}(\mathcal{A})$. This means the following: whenever M, N are isomorphic representations in $\mathfrak{R}(\mathcal{A})$ and M vanishes on I, then the same holds for N.

Proof.– We show by induction on i that $N(I_i) = 0$ for all $0 \leq i \leq t$. This being clear for $i = 0$, we suppose $i > 0$ and $N(I_{i-1}) = 0$. Since N is a functor, we only have to show that $N(b_i) = 0$.

Let $\sigma : V \otimes M \longrightarrow N$ be the isomorphism, with inverse $\sigma' : V \otimes N \longrightarrow M$, which exists by assumption. Then the existence of the grouplike $\omega : A' \longrightarrow V$ implies that

$$\sigma(\omega(1_Y) \otimes \sigma'(\omega(1_Y) \otimes n)) = n$$

for all $n \in N(Y)$ and all indecomposable object Y in A'. Assume $b_i \in I(X, Y)$ and $n \in N(Y)$, we want to see that $b_i n = N(b_i)(n) = 0$. Since $\delta(b_i) = b_i\omega(1_Y) - \omega(1_X)b_i$, we have:

$$b_i n = \sigma(b_i\omega(1_Y) \otimes \sigma'(\omega(1_Y) \otimes n)) = \sigma(\delta(b_i) \otimes \sigma'(\omega(1_Y) \otimes n)) + \sigma(\omega(1_X) \otimes b_i\sigma'(\omega(1_Y) \otimes n)).$$

Now use the compatibility of I with δ, M(I)=0 and our induction hypothesis to finish the proof.

2.– Observe that whenever $\mathcal{A} = (A, V)$ is a bocs and we have functors $\theta : A \longrightarrow B$ and $\eta : B \longrightarrow C$, then the following ***transitivity formula*** holds:

$$(\mathcal{A}^\theta)^\eta \cong \mathcal{A}^{\theta\eta}.$$

Thus given an ideal I of A, if we denote by I_θ the ideal of B generated by $\theta(I)$, $\eta : A \longrightarrow A/I$ and $\nu : B \longrightarrow B/I_\theta$ the canonical projections, we have an induced functor $\bar{\theta} : A/I \longrightarrow B/I_\theta$ with $\eta\bar{\theta} = \theta\nu$. Hence we can identify $(\mathcal{A}^\theta)^\nu$ with $(\mathcal{A}^\eta)^{\bar{\theta}}$. We are interested in some properties of θ^* which will be inherited by the restriction $\bar{\theta}^*$ in the following commutative square:

$$\begin{array}{ccc} \mathfrak{R}(\mathcal{A}^\theta) & \xrightarrow{\qquad\theta^*\qquad} & \mathfrak{R}(\mathcal{A}) \\ \nu^* \uparrow & & \uparrow \eta^* \\ \mathfrak{R}((\mathcal{A}^\theta)^\nu) = \mathfrak{R}((\mathcal{A}^\eta)^{\bar{\theta}}) & \xrightarrow{\quad\bar{\theta}^*\quad} & \mathfrak{R}(\mathcal{A}^\eta). \end{array}$$

The next result is easy to establish using last section:

Lemma .– Assume $\mathcal{A} = (A, V)$ is a bocs with grouplike ω and let I be any ideal of A compatible with ω. With the previous notation, assume N in $\mathfrak{R}(\mathcal{A}^\eta)$ satisfies $N \cong \theta^*(M)$ in $\mathfrak{R}(\mathcal{A})$ for some M in $\mathfrak{R}(\mathcal{A}^\theta)$. Then M is in $\mathfrak{R}((\mathcal{A}^\eta)^{\bar{\theta}})$. In particular, $\bar{\theta}^*$ is dense whenever θ^* is so.

3.– We shall study induced bocses of a given layered bocs $\mathcal{A} = (A, V)$, by functors $\theta : A \longrightarrow B$ of some special types as in [CB]. Thus we need the following:

Lemma.– Assume I is an ideal of A compatible with the grouplike of the layer of $\mathcal{A}$. Let $\theta : A \longrightarrow B$ be a functor of either of the following types:

(a) θ is a "regularization functor", as in [CB;4.2];

(b) θ is the pushout of an admissible functor,as in [CB;4.1,4.3].

Then it is shown in [CB;4.2 and 4.5] that the induced bocs $\mathcal{A}^\theta$ is layered. We claim here that I_θ —the ideal of B generated by $\theta(I)$— is compatible with the induced grouplike.

Proof.– Suppose $L = (A'; \omega; a_1, ..., a_n; v_1, ..., v_m)$ is the layer of $\mathcal{A}$, call δ the differential associated with ω and $b_1, b_2, ..., b_t$ the generators of I providing the compatibility with δ.

case(a): Here B is the subcategory of A generated by A' and $a_2, a_3, ..., a_n$; $\theta : A \longrightarrow B$ acts as the identity on $A', a_2, a_3, ..., a_n$ and sends a_1 to zero.

In this case $\theta(b_1), \theta(b_2), ..., \theta(b_t)$ generate I_θ as a B–B-bimodule. Using these generators, [CB;4.4(1)], $\theta(I_i) \subseteq (I_\theta)_i$ and our assumption on I, one verifies immediately the compatibility of I_θ.

case(b). Here assume that θ is the pushout of the admissible functor $\theta' : A' \longrightarrow B'$. Let $X_\lambda, Y_\lambda, e_\lambda, f_\lambda, \Lambda$ be as in the definition of admissible functor [CB;4.3]. Let $b_{i\lambda\mu} := e_\lambda \theta(b_i) f_\mu$, defined for $b_i \in I(X_\lambda, X_\mu)$, and ordered in such a way that:

$$b_{i\lambda\mu} < b_{i'\lambda'\mu'} \text{ iff } i < i' \text{ or } (i = i' \text{ and } \lambda < \lambda') \text{ or } (i = i', \lambda = \lambda' \text{ and } \mu > \mu').$$

We claim that $\{ b_{i\lambda\mu} \}$ is a set of generators of I_θ which guarantee the compatibility of I_θ with the induced differential δ' of $\mathcal{A}^\theta$. In order to make this evident, expand the formula for the differential δ' (see [CB;proof of 4.5]):

$$\delta'(b_{i\lambda\mu}) = e_\lambda \otimes \delta(b_i) \otimes f_\mu + \sum \tau(e_\lambda \otimes f_\nu) b_{i\lambda\mu} - \sum b_{i\lambda\xi} \tau(e_\xi \otimes f_\mu) \ ,$$

where the first sum runs over indexes with $\nu \neq \lambda$ and $X_\lambda = X_\nu$ and the second with $\xi \neq \mu$ and $X_\xi = X_\mu$.

Here the first term of the right hand side belongs to $e_\lambda I_{i-1} \otimes V \otimes f_\mu + e_\lambda \otimes V \otimes I_{i-1} f_\mu$. From the definition of the order of the chosen set of generators $\{ b_{i\lambda\mu} \}$, our claim follows.

4.–Proposition.– Let $\mathcal{A} = (A, V)$ be a layered bocs with minimal category A'. Let X be an indecomposable object in A and suppose that $A'(X, X) = k[x, f(x)^{-1}]$. Let I be any ideal of A compatible with the layer of $\mathcal{A}$ (i.e. with the grouplike of this layer), and such that $\mathcal{A}_I$ is representation-finite. Then there is a category B and a functor $\theta : A \longrightarrow B$ such that:

(a) $\mathcal{A}^\theta$ is layered and I_θ is compatible with the induced layer;

(b) If B' is the minimal category of the layer of $\mathcal{A}^\theta$, then the number of indecomposable objects in B' with non-trivial endomorphism ring is strictly smaller than that of A';

(c) $\bar{\theta}^* : \Re((\mathcal{A}^\theta)_{I_\theta}) \longrightarrow \Re(\mathcal{A}_I)$ is an equivalence;

(d) If N is a representation in $\Re((\mathcal{A}^\theta)_{I_\theta})$, then $\| N \| \leq \| \theta^*(N) \|$. The inequality is strict whenever $\theta^*(N)(x) \neq 0$.

(Recall that the norm is defined for representations of layered bocses, thus our N is in $\Re(\mathcal{A}^\theta)$ but vanishes on I_θ).

Proof.– First observe that $I(X, X) \cap A'(X, X) \neq 0$. Otherwise, each element in

$$P = \{(m, \lambda)/m \text{ a natural number, } \lambda \in k \text{ not a root of } f\}$$

determines a representation $J = J_{(m,\lambda)}$ defined by $J(X) = k^m$, $J(Y) = 0$ for all indecomposable $Y \neq X$, $J(x)$ is the $m \times m$ Jordan block with eigenvalue λ, and $J(a_i) = 0$ for all i —where $a_1, ..., a_n$ are the free generators of A over A'. We would then have an infinite set $\{J_p\}_{p \in P}$ of non-isomorphic indecomposable representations of $\mathcal{A}_I$, a contradiction.

Consider any monic polynomial $h(x) \in k[x]$ which generates $I(X,X) \cap A'(X,X)$, a principal ideal of $k[x, f(x)^{-1}]$, and decompose it as a product of its distinct linear factors:

$$h(x) = (x - \lambda_1)^{r_1}(x - \lambda_2)^{r_2} \cdots (x - \lambda_s)^{r_s}.$$

After multiplying h by the inverses of some linear factors of f, if necessary, we may assume that h and f have no common root. Now take $g(x) = (x - \lambda_1) \cdots (x - \lambda_s)$.

Observe that given any representation M in $\Re(\mathcal{A}_I)$, with $M(x) \neq 0$, $M(h(x)) = 0$ implies that the eigenvalues of $M(x)$ are included in $\{\lambda_1, ..., \lambda_s\}$. Hence $M(g(x))$ is not invertible whenever $M(x) \neq 0$. Furthermore, if J_{m,λ_i} —the $m \times m$ Jordan block with eigenvalue λ_i— appears in the Jordan block decomposition of the linear map $M(x)$, then $m \leq r_i$. Choose a natural number r larger than all r_i's.

Now follow the proof of [CB;4.7] to get a functor $\psi : A \longrightarrow B$ satisfying:

(i) The induced bocs $\mathcal{A}^{\psi}$ is layered;

(ii) Every representation M in $\Re(\mathcal{A})$ with $M(h(x)) = 0$ is isomorphic to $\psi^*(N)$ for some N in $\Re(\mathcal{A}^{\psi})$; and

(iii) If N is a representation in $\Re(\mathcal{A}^{\psi})$, then $\| N \| \leq \| \psi^*(N) \|$. If $\psi^*(N)$ vanishes on I and not on x, the inequality is strict.

Thus, using Lemma 2, we have an induced equivalence of categories:

$$\bar{\psi}^* : \Re((\mathcal{A}^{\psi})_{I_\psi}) \longrightarrow \Re(\mathcal{A}_I).$$

Observe that $\psi(h(x)) \in I_\psi$ implies $h(y) \in I_\psi$, where $B'(Y,Y) = k[y, f(y)^{-1}, g(y)^{-1}]$, as in the proof of [CB;4.7]. Hence, $h(y)$ is invertible, $1_Y \in I_\psi$ and all representations N of $\Re((\mathcal{A}^{\psi})_{I_\psi})$ satisfy $N(Y) = 0$.

Now if C is the full subcategory of B' whose objects have no direct summand isomorphic to Y, then [CB;4.6] provides us with another functor $\rho : B \longrightarrow C$ which induces a layered bocs $(\mathcal{A}^{\psi})^{\rho} = \mathcal{A}^{\psi\rho}$. Furthermore, ρ^* is a norm-preserving equivalence from $\Re((\mathcal{A}^{\psi})^{\rho})$ to the full subcategory of $\Re(\mathcal{A}^{\psi})$ consisting of representations which are zero on Y. Thus again using 2, we obtain an equivalence:

$$\bar{\rho}^* : \Re((\mathcal{A}^{\psi})^{\rho}_{(I_\psi)_\rho}) \longrightarrow \Re((\mathcal{A}^{\psi})_{I_\psi}).$$

Then $\theta = \psi\rho$ induces an equivalence $\bar{\theta}^* = \bar{\rho}^*\bar{\psi}^*$. Now I_θ is compatible because I, I_ψ are so, and ψ, ρ preserve this property because of 3. Part (d) of our statement is deduced from last considerations and Lemma 2.

5.– We shall find useful some very general considerations on bocses induced by canonical projections:

Lemma.– Suppose $\mathcal{A} = (A, V)$ is a bocs with counit ϵ, comultiplication μ and grouplike $\omega : A' \longrightarrow V$. Let J be any ideal of A and call ν the canonical projection $A \longrightarrow A/J =: \tilde{A}$. Denote by J' the ideal of A' given by $J(X,Y) = J(X,Y) \cap A'(X,Y)$ for all X, Y in A'. Denote

$\tilde{A}' := A'/J'$. Then we have:

(a) The inclusion $i : A' \longrightarrow A$, ϵ, μ, and ω induce an embedding $\tilde{i} : \tilde{A}' \longrightarrow \tilde{A}$, and bimodule morphisms $\tilde{\epsilon} : \tilde{V} \longrightarrow \tilde{A}$, $\tilde{\mu} : \tilde{V} \longrightarrow \tilde{V} \otimes_{\tilde{A}} \tilde{V}$ and $\tilde{\omega} : \tilde{A}' \longrightarrow \tilde{V}$ where $\tilde{V} := V/(JV + VJ)$. Furthermore, $\tilde{\mathcal{A}} := (\tilde{A}, \tilde{V}, \tilde{\epsilon}, \tilde{\mu})$ is a bocs, with grouplike $\tilde{\omega}$, which is isomorphic to $\mathcal{A}^{\nu}$; we identify them often.

(b) If ω is a reflector, then so is $\tilde{\omega}$;

(c) If δ, $\tilde{\delta}$ are the differentials associated with ω, $\tilde{\omega}$ respectively, then $\delta_1 \nu = \nu \tilde{\delta}_1$ and $\delta_2(\nu \otimes \nu) = \nu' \tilde{\delta}_2$, where ν' denotes the restriction of ν to the kernels of ϵ and $\tilde{\epsilon}$.

(d) Assume that $\delta(J) \subseteq JV + VJ$, then the kernel of $\tilde{\epsilon}$ is isomorphic to the $\tilde{A}$–$\tilde{A}$-bimodule $U := \bar{V}/(J\bar{V} + \bar{V}J)$. If moreover, $\bar{V}$ is freely generated as an A–A-bimodule by indecomposable elements $v_1, ..., v_m$, we get U freely generated over $\tilde{A}$ by the non-zero classes modulo $J\bar{V} + \bar{V}J$ of those elements.

(e) If I is any other ideal of A, compatible with ω, then $I_{\nu} = (I + J)/J$ is an ideal of $\tilde{A}$ compatible with $\tilde{\omega}$.

Proof.–To obtain (d), one shows that $\delta(J) \subseteq JV + VJ$ together with $V = \bar{V} \oplus A$ imply that $JV + VJ = (J\bar{V} + \bar{V}J) \oplus J$.

(Some of these statements have already been considered, with a slightly different formulation, in [M]).

6.–Lemma.– Let $\mathcal{A} = (A, V)$ be a bocs with layer $L = (A'; \omega; a_1, ..., a_n; v_1, ..., v_m)$. Suppose that $A'(X, X) = k[x, f(x)^{-1}]$ for some indecomposable object X in A. Assume x is not a factor of $f(x)$ in $k[x]$. Let J be the ideal of A generated by x. Then, with the notation of 5, we have:

(a) $\tilde{L} = (\tilde{A}'; \tilde{\omega}; \tilde{a}_1, ..., \tilde{a}_n; \tilde{v}_1, ..., \tilde{v}_m)$ is a layer for $\tilde{\mathcal{A}}$, where $\tilde{a}_i$, $\tilde{v}_i$ denote classes modulo J, $JV + VJ$ respectively.

(b) If $\nu : A \longrightarrow \tilde{A}$ is the canonical projection and N is a representation in $\Re(\mathcal{A}^{\nu})$ such that $\nu^*(N)(X) \neq 0$, then $\| N \| < \| \nu^*(N) \|$.

(If x is a factor of $f(x)$ in $k[x]$, then a similar statement holds, but we have to eliminate from $\tilde{L}$ all $\tilde{a}_i$ and $\tilde{v}_i$ which start or stop at X. See [CB;4.6]).

Proof.– Using Leibnitz rule and $\delta(x) = 0$, one obtains $\delta(J) \subseteq JV + VJ$. Thus (d) of last Lemma applies.

7.– By definition, a given layered bocs $\mathcal{A} = (A, V)$ with minimal category A' is called *trivial* iff A' is trivial and $A = A'$.

Theorem.– Let $\mathcal{A} = (A, V)$ be a layered bocs and I any ideal of A such that $\mathcal{A}_I$ is representation-finite and I is compatible with the grouplike of $\mathcal{A}$. Then there is a trivial category B and a functor $\theta : A \longrightarrow B$ such that $\mathcal{A}^{\theta}$ is a trivial bocs and

$$\bar{\theta}^* : \Re((\mathcal{A}^{\theta})_{I_\theta}) \longrightarrow \Re(\mathcal{A}_I) \text{ is an equivalence.}$$

Proof.– We shall give a procedure to construct, given a non-trivial layered bocs $\mathcal{A}$ as in the assumptions of the theorem, a new one $\mathcal{A}^\theta$, induced from $\mathcal{A}$ by some funtor $\theta : A \longrightarrow B$, which satisfies the same assumptions —now with the ideal I_θ. Moreover, θ will induce an equivalence $\bar{\theta}^*$ as in the statement of our theorem. After some steps, this procedure will reduce the "complexity of the bocs" in the following sense:

If $M_1, ..., M_s$ is a set of representatives of the isoclasses of indecomposable representations of $\Re(\mathcal{A}_I)$, then define the *complexity* of $\mathcal{A}_I$ as $c(\mathcal{A}_I) :=\| \oplus M_i \|$, the parallels denote the norm in $\Re(\mathcal{A})$.

Let $L = (A'; \omega; a_1, ..., a_n; v_1, ..., v_m)$ be the layer of $\mathcal{A}$.

step 1.– Assume $A'(X, X) = k[x, f(x)^{-1}]$ for some indecomposable X in A'.

case 1.1: $1_X \in I(X, X)$. Here we have $M(X) = 0$ for all M in $\Re(\mathcal{A}_I)$. Thus we apply [CB;4.6] to the subcategory $\{X\}$ of A, and get a functor $\theta : A \longrightarrow B$ and a layered bocs $\mathcal{A}^\theta$ such that $\bar{\theta}^* : \Re((\mathcal{A}^\theta)_{I_\theta}) \longrightarrow \Re(\mathcal{A}_I)$ is a norm-preserving equivalence. Thus $\bar{\theta}^*$ preserves the complexity and there are less indecomposable objects in B than in A.

case 1.2: $1_X \notin I(X, X)$ but $x \in I(X, X)$. Here we apply Lemma 6 to induce from $\nu : A \longrightarrow A/J$ an equivalence $\bar{\nu}^* : \Re((\mathcal{A}^\nu)_{I_\nu}) \longrightarrow \Re(\mathcal{A}_I)$. The pair $(\mathcal{A}^\nu, I_\nu)$ satisfies the assumptions of the theorem because of Lemmas 5 and 6. Since $M(X) \neq 0$ for some indecomposable M in $\Re(\mathcal{A}_I)$, the same holds for some $\bar{\nu}^*(N)$ with N indecomposable in $\Re((\mathcal{A}^\nu)_{I_\nu})$. Then 6-(c) implies that $c((\mathcal{A}^\nu)_{I_\nu}) < c(\mathcal{A}_I)$.

case 1.3: $x \notin I(X, X)$. Then $M(x) \neq 0$ for some M in $\Re(\mathcal{A}_I)$. Here we apply Proposition 4 to get the desired functor $\theta : A \longrightarrow B$ and pair $(\mathcal{A}^\theta, I_\theta)$, with $c((\mathcal{A}^\theta)_{I_\theta}) < c(\mathcal{A}_I)$. The last inequality follows from the fact that $J = (x)$ is compatible, $\bar{\theta}^*$ is dense and Lemmas 4(d) and 1.

At this point, after step 1 has been performed, we have either reduced the complexity of the bocs or reduced the number of indecomposable objects in A. After applying step 1 to each indecomposable object X in A' with non-trivial endomorphism ring, we may assume $A'(X, X) = k$ for all indecomposable X.

step 2.– Assume $\delta(a_1) = 0$ and $a_1 \in A(X, X)$ for some X. Here we first consider A'', the subcategory of A generated by A' and a_1, which is minimal (see [CB;5]), and replace the layer of $\mathcal{A}$ by the new one $(A''; \omega''; a_2, ..., a_n; v_1, ..., v_m)$. Now go back to step 1 to destroy the non-trivial part of $A''(X, X)$ and with this eliminate one indecomposable object of A or decrease the complexity of the bocs.

After applying step 2 several times, if necessary, we may assume that either $\mathcal{A}$ is already trivial or we are in the situation of the next step.

step 3.– $A'(Z, Z) = k$ for all indecomposable Z and $a_1 \in A(X, Y)$ for some different indecomposables X, Y.

If $1_X \in I(X, X)$ (or $1_Y \in I(Y, Y)$) then we eliminate X (resp. Y) as in case 1.1 of step 1, and obtain a norm-preserving equivalence and a category with less indecomposable objects. Otherwise, we have $\oplus M_i(X) \neq 0$ and $\oplus M_i(Y) \neq 0$. We consider two possibilities:

case 3.1: $\delta(a_1) = 0$. Here we apply [CB;4.9], together with Lemma 2, to get the desired equivalence which decreases the complexity.

case *3.2:* $\delta(a_1) \neq 0$. Then $\delta(a_1) = \lambda_1 v_1 + \cdots + \lambda_m v_m$ with $\lambda_1, ..., \lambda_m \in k$. Hence, after a suitable change of basis in the layer (see [CB;proof of Thm.A,case(b)]), we can apply [CB;4.2], together with Lemma 2 to obtain the required equivalence which decreases complexity.

In conclusion, we have described a procedure which must stop after a finite number of steps because we can not reach a category B without objects or a bocs $(\mathcal{A}^\theta)_{I_\theta}$ with zero complexity. Thus we must stop at the beginning of step 3 with a trivial bocs, as wanted.

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