Werk

Titel: P-Amenable Locally Compact Groups.

Autor: Ganesan, S.

Jahr: 1986

PURL: https://resolver.sub.uni-goettingen.de/purl?362162050_0101|log29

Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

info@digizeitschriften.de

Mh. Math. 101, 301–308 (1986)

Monatshefte für
Mathematik
© by Springer-Verlag 1986

P-Amenable Locally Compact Groups

By

S. Ganesan, Portland*

(Received 19 August 1985; in revised form 23 December 1985)

Abstract. Let G denote a locally compact group and $P(G)$ $[P_c(G)]$ the topological semigroup of absolutely continuous [absolutely continuous and compactly supported] probability measures on G. We say that G is P-amenable [P_c-amenable] if the topological semigroup $P(G)[P_c(G)]$ is amenable. Some combinatorial properties of this class of groups are studied. The relationship between amenability and P-amenability of G is investigated. It is shown that for a connected solvable P_c-amenable group G, G has polynomial growth if, and only if, $P_c(S) = \{f \in P_c(G) \mid \operatorname{supp} f \subset S\}$ is amenable for any open subsemigroup S of G.

1. P-Amenability

Let S be a topological semigroup (i.e., S is a semigroup with a Hausdorff topology such that the mapping from $S \times S$ to S defined by $(s, t) \to st$ for all s, t in S is continuous when $S \times S$ has the product topology). Let X be a closed subspace of $B(S)$, the space of all bounded real-valued functions defined on S equipped with the supremum norm. Assume that X contains the constant functions. A linear functional μ on X is a mean if μ is positive and of norm one. If for each f in X, ${}_sf$ is also in X, where ${}_sf(t) = f(st)$ for each s, t in S and if $\mu({}_sf) = \mu(f)$ for each f in X and s in S, then μ is called a left invariant mean (LIM) on X. If G is a locally compact group, G is said to be *amenable* (more accurately, left amenable) if there is a LIM on the space of bounded continuous functions on G. A function f in $B(S)$ is left uniformly continuous if f is continuous and whenever s_α converges to s where s_α, s in S, $\|{}_{s_\alpha}f - {}_sf\|_\infty$ converges to zero. Let $LUC(S)$ denote the set of all left uniformly continuous functions on S. $LUC(S)$ is a closed subspace of $B(S)$. Let G be a locally compact group with a fixed left Haar measure. We denote it by dx. Let $P(G)$ $[P_c(G)]$ denote the set

* This paper contains a part of the author's doctoral thesis at the State University of New York at Albany. The author wishes to thank Professor JOE JENKINS for his valuable suggestions and encouragement during the course of this work.

of absolutely continuous [compactly supported and absolutely continuous] probability measures on G. $P(G)$ and $P_c(G)$ are topological semigroups under the convolution product in $L'(G)$ equipped with the norm topology.

Definition. Let G denote a locally compact group. We say that G is P-amenable [P_c-amenable] if $P(G)$ [$P_c(G)$] as a topological semigroup is left amenable, i.e., there exists a left invariant mean on the space of bounded continuous functions on $P(G)$ [$P_c(G)$].

Remark 1.2. a) If G is an abelian group, then $P(G)$ is an amenable semigroup.

b) Suppose that G is compact and assume the Haar measure is normalized. Then, $\chi_G \in P(G)$ and hence the point evaluation e_G, defined by $e_G(F) = F(\chi_G)$ for each F in $CB(P(G))$ is a LIM on $CB(P(G))$, the continuous bounded functions on $P(G)$.

c) $P^+(G) = \{f \in L'(G) | f \geqslant 0 \text{ and } \int_G f(x)\,dx > 0\}$ is a topological semigroup under the convolution product in $L'(G)$.

Remarks 1.2a and 1.2b show that abelian and compact groups are P-amenable.

Proposition 1.3. *Let G be a locally compact group. G is P-amenable if and only if $P^+(G)$ is an amenable semigroup.*

Proof. Assume that $P^+(G)$ is amenable. Let $\pi: P^+(G) \to P(G)$ be defined by $\pi(f)(x) = f(x)/\int_G f(x)\,dx$, for $f \in P^+(G)$. Then π is an onto homomorphism. It follows from [1] that $P(G)$ is amenable. Now assume that $P(G)$ is amenable. Let m be a LIM on $CB(P(G))$. Let $\theta \in CB(P^+(G))$ and define $\tilde{\theta}$ on $P^+(G)$ by $\tilde{\theta}(g) = \langle m, {}_g\theta|_{P(G)}\rangle$ where ${}_g\theta(f) = \theta(g * f)$. If $f \in P(G)$, $\tilde{\theta}(g * f) = \langle m, {}_{g*f}\theta\rangle = \langle m, {}_f({}_g\theta)\rangle = \langle m, {}_g\theta\rangle = \tilde{\theta}(g)$. Also note $({}_f\theta)^{\sim} \equiv {}_f(\tilde{\theta})$, for $f \in P^+(G)$. Let e_λ be an approximate identity in $P(G)$. Then for $f \in P^+(G)$, $\tilde{\theta}(f) = \lim_\lambda \tilde{\theta}(e_\lambda * f) = \lim_\lambda \tilde{\theta}(f(G)\, e_\lambda * f/f(G)) = \lim_\lambda \tilde{\theta}(f(G)\, e_\lambda)$, since $f/f(G) \in P(G)$ where $f(G) = \int_G f(x)\,dx$. This shows for any $\alpha > 0$ $\lim_\lambda \tilde{\theta}(\alpha\, e_\lambda)$ exists. Define $\bar{\theta}$ on $\mathbb{R}^+$ by $\bar{\theta}(\alpha) = \lim_\lambda \tilde{\theta}(\alpha\, e_\lambda)$ and let $\bar{m}$ be a LIM on $B(\mathbb{R}^+)$. Define $\bar{\mu}$ on $CB(P^+(G))$ by $\langle \bar{\mu}, \theta\rangle = \langle \bar{m}, \bar{\theta}\rangle$. Clearly $\bar{\mu}$ is a mean and is left invariant. For any $f, f_1 \in P^+(G)$,

$$({}_f\theta)^{\sim}(f_1) = {}_f(\tilde{\theta})(f_1) = \lim_\lambda \tilde{\theta}(e_\lambda * f * f_1) = \lim_\lambda \tilde{\theta}(f_1(G)\, e_\lambda * f * f_1/f_1(G)) =$$

$$= \lim_{\lambda} \tilde{\theta}(f_1(G)\, e_\lambda * f) = \lim_{\lambda} \tilde{\theta}(f_1(G) f(G)\, e_\lambda * f/f(G)) =$$

$$= \lim_{\lambda} \tilde{\theta}(f_1(G) f(G)\, e_\lambda).$$

In particular, $({}_f\theta)^{\sim}(a\, e_\lambda) = \lim_{\lambda} \tilde{\theta}(a f(G)\, e_\lambda)$. Hence

$$(\overline{{}_f\theta})(a) = \lim_{\lambda} ({}_f\theta)^{\sim}(a\, e_\lambda) = \lim_{\lambda} \tilde{\theta}(a f(G)\, e_\lambda) =$$

$$= \bar{\theta}(a f(G)) = \bar{\theta}(f(G)\, a) = {}_{f(G)}[\bar{\theta}](a),$$

i.e., $(\overline{{}_f\theta}) \equiv {}_{f(G)}(\bar{\theta})$. Therefore,

$\langle \bar{\mu}, {}_f\theta \rangle = \langle \bar{m}, (\overline{{}_f\theta}) \rangle = \langle \bar{m}, {}_{f(G)}[\bar{\theta}] \rangle = \langle \bar{m}, \bar{\theta} \rangle = \langle \bar{\mu}, \theta \rangle.$

This proves $\bar{\mu}$ is a *LIM* on $CB(P^+(G))$.

We make use of the above proposition and prove the following.

Proposition 1.4. *Any quotient of a P-amenable group is P-amenable. Any discrete subgroup of a discrete P-amenable group is P-amenable.*

Proof. Let G' be a homomorphic image of a P-amenable group G. Let π be an onto homomorphism of G into G'. Let $N = \ker \pi$. Then $G/N \simeq G'$ and it is enough to prove that G/N is P-amenable. Let m be a *LIM* on $CB(P^+(G))$. Define $\bar{m}$: $CB(P^+(G/N)) \to \mathbb{R}$ by $\bar{m}(F) = m(\bar{F})$ where $\bar{F}: P^+(G) \to \mathbb{R}$ is defined by $\bar{F}(f) = F(T_N f)$ where $T_N f(\xi) = \int_N f(\xi x)\, dx$. Clearly $\bar{F}$ is a bounded continuous function and $\bar{m}$ is a mean. Let g be in $P^+(G/N)$. Since $T_N: L'(G) \to L'(G/N)$ is onto, we can find a $f \in P^+(G)$ such that $T_N f = g$. Now, observe that

$${}_g\bar{F}(h) = {}_gF(T_N h) = F(g * T_N h) = F(T_N f * T_N h) =$$

$$= F(T_N(f * h) = \bar{F}(f * h) = {}_f\bar{F}(h).$$

Hence, $\langle \bar{m}, {}_gF \rangle = \langle m, {}_g\bar{F} \rangle = \langle m, {}_f\bar{F} \rangle = \langle m, \bar{F} \rangle = \langle \bar{m}, F \rangle$ which proves $\bar{m}$ is a left invariant mean on $CB(P^+(G/N))$.

Let $G = \bigcup_{a} H t_a$ where $H t_a \cap H t_\beta = \emptyset$ for $a \neq \beta$. For $\varphi \in P(G)$ write $\varphi \equiv \Sigma(\varphi_a)_{t_a}$ and $\varphi_a \in P^+(H)$. For $\theta \in B(P^+(H))$ define $\bar{\theta} \in B(P(G))$ as $\bar{\theta}(\varphi) = \sum \|\varphi_a\|\, \theta(\varphi_a)$ where $\varphi \in P(G)$ and $\varphi = \Sigma(\varphi_a)_{t_a}$. Let m be a *LIM* on $B(P(G))$. Define a linear functional $\bar{m}$, on $B(P^+(H))$ by $\langle \bar{m}, \theta \rangle = \langle m, \bar{\theta} \rangle$ for $\theta \in B(P^+(H))$. Clearly $\bar{m}$ is linear and positive. If $\theta \equiv 1$ then $\bar{\theta}(\varphi) = \sum \|\varphi_a\| = 1$ for all $\varphi \in P(G)$. Let $\psi \in P^+(H))$, $\theta \in B(P^+(G))$, and $\beta^{-1} = (\int_H \psi(x)\, dx)$; then $\beta\psi \in P(G)$. Let $\varphi \in P(G)$ and $\varphi = \sum (\varphi_a)_{t_a}$. Now, ${}_{\beta\psi}[\bar{\theta}](\varphi) = \bar{\theta}(\beta\psi * \varphi) = \sum \|\beta\psi * \varphi_a\|\, \theta(\beta\psi * \varphi_a) =$

$= \sum \|\varphi_a\|_{\beta\psi}\theta(\varphi_a) = [\overline{{}_{\beta\psi}\theta}](\varphi)$ and $\langle \bar{m}, {}_{\psi}\theta\rangle = \langle m, \bar{\theta}\rangle$ for all $\psi \in P(H)$ and $\theta \in B(P^+(H))$. Now define m_1, a linear functional on $B(P(H))$, by $\langle m_1, \theta\rangle = \langle \bar{m}, \tilde{\theta}\rangle$ where $\tilde{\theta} \in B(P^+(H))$ is defined by $\tilde{\theta}(f) = \theta(f/\int_G f(x)\,dx)$. Clearly, ${}_{\psi}(\tilde{\theta}) = ({}_{\psi}\theta)^{\sim}$ for $\psi \in P(H)$ and it is easy to verify that m_1 is a *LIM* on $B(P(H))$.

Proposition 1.5. *Let K be a compact normal subgroup of G and assume that there exists a left invariant mean on $B(P(G/K))$. Then there exists a left invariant mean on $B(P(G))$.*

Proof. Let I denote the right ideal in $P(G)$ which consists of all functions which are constant on cosets. It is enough to prove that I is a left amenable semigroup. Let θ be a bounded real valued function on I. Define $\bar{\theta}$: $P(G/K) \to \mathbb{R}$ by $\bar{\theta}(\bar{f}) = \theta(\bar{f} \circ \Pi_K)$ where Π_K: $G \to G/K$ is the canonical map. Now, $\bar{f} \circ \Pi_K$ is constant on cosets. Since K is compact and $\bar{f} \in P(G/K)$ we can show using extended Weil's formula, $\bar{f} \circ \Pi_K \in I$. Hence, $\bar{\theta}$ is well defined. Let $\bar{m}$ be a *LIM* on $B(P(G/K))$ and define a linear functional m on $B(I)$ by $\langle m, \theta\rangle = \langle \bar{m}, \bar{\theta}\rangle$. Clearly, m is a mean on $B(I)$. Note that if $f \in I$ then $f = \bar{f} \circ \Pi_K$ for some $\bar{f} \in P(G/K)$. It is easy to verify that $(\overline{{}_f\theta})(g) = {}_{\bar{f}}[\bar{\theta}](g)$ for any $g \in P(G/K)$. Hence, $\langle m, {}_f\theta\rangle = \langle m, \theta\rangle$ for any $f \in I$ which proves that m is a *LIM* on $B(I)$.

The above proposition gives a non-abelian non-compact P-amenable group as seen in the following example.

Example 1.6. Let H be the Heisenberg group, i.e.,

$$H = \left\{ \begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix} : x, y, z \text{ are real} \right\} \text{and } N = \left\{ \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : n \text{ is an integer} \right\}.$$

Let G be the quotient group H/N. Then the centre $C(G)$ of G is isomorphic to T, the circle group and $C(G) = G'$, the commutator subgroup of G. Since $G/C(G)$ is abelian and $C(G)$ is a compact normal subgroup, by the above proposition, G is a connected P-amenable group.

Remark 1.7. a) As in GREENLEAF [3, Theorem 1.2.7] one can show that if G is a discrete group which is a directed union of P-amenable subgroups H then G is P-amenable.

b) Suppose that m is a *RIM* on $CB(S)$, where $S = P(G)$ then there exists a *LIM* on $CB(S)$. For, given $\theta \in CB(S)$ define a linear functional $\bar{m}$ on $CB(S)$ by $\langle \bar{m}, \theta\rangle = \langle m, \bar{\theta}\rangle$ where $\bar{\theta}(f) = \theta(f^*)$. It can be verified that $\bar{m}$ is a *LIM* on $CB(S)$.

c) It is easily shown that existence of a *RIM* on $LUC(S)$, provides a two-sided invariant mean.

2. Additively Uniformly Continuous Functions on $P(G)$

In this section, we investigate the relation between amenable and P-amenable groups. We introduce the function space $AUC(P(G))$ (the additively uniformly continuous functions θ on $P(G)$), the space of continuous functions on $P(G)$) such that given $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $\|f - g\| < \delta$, $|\theta(f) - \theta(g)| < \varepsilon$ for $f, g \in P(G)$. We prove that G is amenable if, and only if, there exists a *RIM* on $AUC(P(G))$. In the next section, we prove that it is equivalent to a Fixed Point Theorem for $P(G)$ under certain actions.

Lemma 2.1. *$AUC(S) \subseteq LUC(S)$ where $S = P(G)$.*

Proof. Let $\theta \in AUC(S)$ and fix $s_0 \in S$. Let $\varepsilon > 0$ be given. We need to show that there exists a nbd V_δ of s_0 such that whenever $s \in V_\delta$ $\|{}_{s_0}\theta - {}_s\theta\| < \varepsilon$. Given $\varepsilon > 0$, choose $\delta > 0$ using continuity of θ. Let $U = \{(s,t) \mid |\theta(s_0 t) - \theta(s t)| < \varepsilon\}$. Let $V_\delta = \{s \in S \mid \|s - s_0\| < \delta/2\}$ Then V_δ is an open set containing s_0 and $V_\delta \times S \subseteq U$. To see this, let $s \in V_\delta$, $t \in S$ then $\|s_0 * t - s * t\| = \|(s_0 - s) * t\| \leqslant \|s_0 - s\| < \delta/2$, hence $|\theta(s_0 * t) - \theta(s * t)| < \varepsilon$ i.e., $V_\delta \times S \subseteq U$. Since, $t \in S$ is arbitrary $\|{}_{s_0}\theta - {}_s\theta\| < \varepsilon$, which proves the lemma.

Theorem 2.2. *Let G be a locally compact group. G is amenable if, and only if, there exists a RIM on $AUC(S)$.*

Proof. Assume that G is amenable, then by [3, Theorem 2.4.2] there exists a net (φ_α) in $P(G)$ such that $\|\varphi * \varphi_\alpha - \varphi_\alpha\| \to 0$ for all $\varphi \in P(G)$. Define, for each α, μ_α by $\mu_\alpha(\theta) = \theta(\varphi_\alpha)$ where $\theta \in AUC(S)$. Clearly each μ_α is a mean on $AUC(P(G))$. Let μ be defined on $AUC(P(G))$ by $\mu(\theta) = \text{weak}^* - \lim \mu_\alpha(\theta)$. It can be easily verified that μ is a *LIM* and hence *RIM* exists.

Conversely, assume that there exists a *RIM* m on $AUC(P(G))$ we will show that there exists a topologically *RIM* on $CB(G)$. Let θ be a continuous bounded realvalued function on G. Define $\bar{\theta}: P(G) \to \mathbb{R}$ by $\bar{\theta}(s) = (\theta * \tilde{s})(e)$, where $\tilde{s}(t) = s(t^{-1})$. Note that $|\bar{\theta}(s_1) - \bar{\theta}(s_2)| = |\theta * (\tilde{s}_1 - \tilde{s}_2)(e)| \leqslant \|s_1 - s_2\| \ \|\theta\|$, which proves $\bar{\theta} \in AUC(P(G))$. Define a linear functional $\bar{m}$ on $CB(G)$ by $\bar{m}(\theta) = \langle m, \bar{\theta} \rangle$ for $\theta \in CB(G)$. Clearly, $\bar{m}$ is a mean. If $p \in S$ and $\theta \in CB(G)$ then $\langle \bar{m}, \theta * \tilde{p} \rangle = \langle m, \overline{\theta * \tilde{p}} \rangle$. On the other hand, $[\bar{\theta}]_p(s) = \bar{\theta}(s * p) = (\theta * (s * p)^{\sim})(e) = (\theta * \tilde{p} * \tilde{s})^{\sim}(e) = \overline{\theta * \tilde{p}}(s)$. So $\langle \bar{m}, \theta * \tilde{p} \rangle = \langle m, \overline{\theta * \tilde{p}} \rangle = \langle m, [\bar{\theta}]_p \rangle = \langle m, \bar{\theta} \rangle = \langle \bar{m}, \theta \rangle$. Hence, $\bar{m}$ is a topological right invariant mean.

Corollary 2.3. *F_2, the free group on two generators is not P-amenable.*

Proof. F_2 is not amenable [3, example 1.2.3].

Remark 2.4. In the proof of Theorem 2.2, one may notice the shift from left-invariance to right-invariance. The computations in the proof are easier to work with right multiplication than with the left and hence the shift.

A Fixed Point Theorem. We first start with the following definition. Let S denote $P(G)$. We say that an action of S on a separated locally convex space X is uniform if for each $y \in X$, $\psi_y: S \to X$, given by $s \to s \cdot y$ is uniformly continuous, i.e., given $\varepsilon > 0$, there exists a $\delta > 0$ such that whenever $\| s - t \| < \delta, p(s \cdot y - t \cdot y) < \varepsilon$ for all semi-norms p which generate the topology on X.

The following two lemmas are motivated from MITCHELL [7] whose proofs are left to the reader.

Lemma 2.5. *Let S act uniformly on X, a compact subset of a normed linear space. Let (s_n), (t_n) be two sequences in S such that $\| s_n - t_n \|$, tends to zero as n tends to infinity. Then, for any fixed $y \in X$, we can choose subsequences $(s_{n_k} \cdot y)$ and $(t_{n_k} \cdot y)$ such that $(s_{n_k} \cdot y)$ and $(t_{n_k} \cdot y)$ have the same limit point.*

Lemma 2.6. *If an action of S on a compact convex set Y, of a separated locally convex space is separately continuous and uniform, then $T_y(C(Y)) \subseteq AUC(S)$, where $(T_y f)(s) = f(s \cdot y)$ for all $s \in S$, $f \in C(Y)$ and $y \in Y$ is fixed.*

Theorem 2.7. *Let $S = P(G)$. The following are equivalent.*

1. *Whenever S acts affinely on a convex compact subset Y of a separated linear space, where the action $S \times Y$ is separately continuous and uniform then Y contains a common fixed point x_0, fixed under this action.*

2. *$AUC(S)$ has a LIM.*

Proof. Using the above lemmas and modifying slightly the proof of Theorem 2 in MITCHELL [8], the above theorem is proved.

Remark 2.8. We would like to point out that whether every amenable group is P-amenable is not known. But, there is an amenable group, for which $P_c(G)$ is not an amenable discrete

semigroup under convolution product. To see this, consider the discrete group $G = Z_2 * Z_2$ (the free product of Z_2 with Z_2), and let $\sigma(G)$ be the semigroup of non-empty compact subsets of G. JENKINS [5] has shown that the discrete semigroup $\sigma(G)$ is not amenable. Note that if f and g are positive functions in $L'(G)$ then $\operatorname{supp}(f * g) = (\operatorname{supp} f) \times \times (\operatorname{supp} g)$. Consider the map h from $P_c(G)$ into $\sigma(G)$ given by $h(f) = \operatorname{supp} f$. Clearly h is an onto semigroup homomorphism. Since $\sigma(G)$ is not amenable, $P_c(G)$ is not a discrete amenable semigroup.

3. Connections Between P-Amenable Groups and Groups with Polynomial Growth

In this section it will be shown that for a connected solvable P_c-amenable group G, polynomial growth is equivalent to $P_c(S)$ is amenable for any open subsemigroup S of G. Let us first recall the definition of groups with polynomial growth. Given a locally compact group G, and a measurable subset U, denote by $|U|$ the left Haar measure of U, and for a positive integer n let $U^n = \{u_1 u_2 u_3 \ldots u_n \mid u_i, 1 \leqslant i \leqslant n\}$. G is said to have polynomial growth if for any compact nbd U of the identity in G, there exists a polynomial p such that $|U^n| \leqslant p(n)$ for all $n = 1, 2, \ldots$. It is shown in JENKINS [6] that for a connected group, G, polynomial growth is equivalent to every open subsemigroup of G is amenable. Note that since $P(G)$ is a metric space, it is not amenable if it has two disjoint closed right ideals.

Lemma 3.1. *Let G be a group. If T is a subsemigroup of G with two disjoint right ideals, then $S = \{f \in P(G) \mid \operatorname{supp} f \subset T\}$ is not amenable.*

Proof. Let I, J be two disjoint right closed ideals in T. Let $I_S = \{f \in S \mid \operatorname{supp} f \subset I\}$ and $J_S = \{f \in S \mid \operatorname{supp} f \subset J\}$. Clearly I_S and J_S are two disjoint closed right ideals in $P(G)$ and hence it is not amenable.

Theorem 3.2. *Let G be a connected solvable P_c-amenable group. Then G has polynomial growth if and only if $P_c(S) = \{f \in P(G) \mid \operatorname{supp} f \subset S\}$ is amenable for any open subsemigroup S of G.*

Proof. If G does not have polynomial growth then G has an open subsemigroup S with two disjoint closed right ideals. Lemma 3.1 contradicts the assumption that $P_c(s)$ is amenable.

For the converse, let m be a LIM on $CB(P_c(G))$. Let S be any open subsemigroup of G. S generates G. Since G is a connected group with polynomial growth, G is ARCHIMEDEAN [2, Proposition 8] i.e., given any compact subset K of G, there exists $s \in S$ such that $s \cdot K \subset S$. We identify each $\varphi \in P_c(G)$ with the linear functional δ_φ on $CB(P_c(G))$ where $\langle \delta_\varphi, \theta \rangle = \theta(\varphi)$ for $\theta \in CB(P_c(G))$. Each δ_φ is a mean and since the set of finite convex sums of such means is weak*-dense in the set of all means on $CB(P_c(G))$, we can choose $\varphi_n \in P_c(G)$ such that $m = \text{weak*} - \lim \varphi_n$. Each φ_n has compact support, hence for each n, there exists a s_n in S such that $s_n \cdot (\operatorname{supp} \varphi_n) \subset S$ i.e., ${}_{s_n}\varphi_n \in P_c(S)$. Let $\mu = \text{weak*} - \lim {}_{s_n}\varphi_n$, then since $\varphi_n \in P_c(G)$ for each n, we have $\langle \mu, \chi_S \rangle = \text{weak*} - \lim \langle {}_{s_n}\varphi_n, \chi_S \rangle = 1$. This proves that the mean μ is a LIM on $CB(P_c(S))$.

Acknowledgement. The author would like to thank the referee for suggestions and corrections he made on the original manuscript.

References

[1] DAY, M. M.: Semigroups and amenability. In: Semigroups. K. W. Folley (Ed. New York: Academic Press. 1969.

[2] GANESAN, S., JENKINS, J. W.: An Archimedian property for groups with polynomial growth. Proc. Amer. Math. Soc. **88**, 550—554 (1983).

[3] GREENLEAF, F. P.: Invariant Means on Topological Groups. Princeton: Van Nostrand. 1969.

[4] JENKINS, J. W.: Amenable subsemigroups of a locally compact group. Proc. Amer. Math. Soc. **25**, 766—770 (1970).

[5] JENKINS, J. W.: Sigma-amenable locally compact groups. Proc. Amer. Math. Soc. **28**, 621—626 (1971).

[6] JENKINS, J. W.: Folner's condition for exponentially bounded groups. Math. Scand. **28**, 165—174 (1974).

[7] MITCHELL, T.: Constant functions and left invariant means on semigroups. Trans. Amer. Math. Soc. **119**, 244—261 (1965).

[8] MITCHELL, T.: Topological semigroups and fixed points. Ill. J. Math. **114**, 630—641 (1970).

S. GANESAN
University of Portland
Department of Mathematics
5000 North Willamette Blvd.
Portland, OR 97203-5798, U.S.A.